The magnetic field strength is thus inhomogeneous, since its value at any point is inversely proportional to the distance R of that point from the major axis of the torus.

4.35. The total drift velocity of a charged particle, perpendicular to the field lines, is the sum of that due to inhomogeneity of the magnetic field and that due to the curvature of the lines of force. The former contribution is given by equation (4.16), with $\nabla_{\perp}B/B$ equal to $1/R$, since BR is constant, and the latter by equation (4.19), so that

$$|v_d| = \frac{W_{\perp}c}{eBR} + \frac{2W_{\parallel}c}{eBR} = \frac{c(W_{\perp} + 2W_{\parallel})}{eBR}.$$

For a Maxwellian distribution of velocities, $W_{\perp}$ is equal to kT and $W_{\parallel}$ to $\frac{1}{2}kT$ at the kinetic temperature T; hence, the drift velocity may be represented by

$$|v_d| = \frac{2ckT}{eBR}. \tag{4.22}$$

4.36. It should be noted that this expression gives only the absolute magnitude of the total drift velocity. Since the drift directions due to both inhomogeneity and curvature have the same dependence on the sign of the charged particle, it can be readily seen that ions will tend to drift in one direction, perpendicular to the plane of the torus, and electrons will tend to drift in the opposite direction. The resulting space charge will produce an electric field which, acting in conjunction with the magnetic field, will cause the plasma as a whole to drift in a direction perpendicular to both fields, as in §4.19. There is no simple relationship between the drift velocity of the plasma as a whole and the individual particle drift velocities, given by equation (4.22). However, since the magnitude of the space-charge electric field must be related to v_d, it would appear that the plasma drift in a toroidal magnetic field can be reduced by increasing the magnetic field strength and the major radius of the torus.

COLLISION PHENOMENA IN A PLASMA

INTRODUCTION

4.37. In the preceding discussion it has been supposed that collisions among the charged particles are rare; that is to say, the collision mean free path is long compared with the dimensions of the confining field, so that the single-particle picture of a plasma is valid. In the present section various aspects of the collision behavior of charged particles will be examined. Estimates will be made of the mean free path for collisions and the exchange of energy accompanying collisions between charged particles. Scattering collisions and energy changes may be regarded as having a perturbing effect on the single-particle behavior in electromagnetic fields.

4.38. The collision between charged particles differs in a highly important respect from that between neutral particles or between a neutral and a charged particle. In the latter cases, there is a fairly definite collision diameter; whenever two particles are within this distance from each other a collision will have occurred. Any approach of the two particles at distances greater than the collision diameter will not result in any interaction. With two charged particles, on the other hand, the situation is very different because the effective range of the Coulomb force, upon which scattering collisions depend, is infinite. In considering charged-particle collisions, it is necessary first to define exactly what type of encounter is to be regarded as a collision. This is generally taken as the interaction which will lead to a deflection (or scattering) through a large angle, namely, 90° or more.

4.39. For simplicity of treatment, the collisions are divided into two categories, although no such distinction actually exists in a plasma. In the first category are the *short-range encounters* (or close collisions) which lead to a scattering angle of 90° or more in a single interaction between a pair of charged particles. The second type is that of *long-range encounters* (or distant collisions); these represent the multiple interactions of a single particle with many other particles such that the net effect is to give a large-angle scattering, i.e., about 90°. In principle, these long-range encounters can extend over the whole distance over which the Coulomb forces are effective, i.e., the whole of the plasma. However, in order to make possible the calculation of the cross section (or equivalent mean free path) for distant collisions of the type just defined, it is necessary to choose a characteristic distance, called the Debye shielding length, within which interaction of a given charged particle with other charged particles may be supposed to occur. Beyond this distance, the plasma may be regarded as being electrically neutral, both macroscopically and microscopically, so that the particle under consideration is not affected by Coulomb forces [2, 7, 8, 12].

ELECTRICAL NEUTRALITY OF A PLASMA

4.40. A basic property of a plasma, which is a consequence of long-range collective interactions among the charged particles, is the tendency toward electrical neutrality. If, over a relatively large volume of the plasma, the density of electrons should differ appreciably from the positive ion density, large electrostatic forces will come into play. As a result, the charged particles will move rapidly in such a manner as to approach a condition of charge equality.

4.41. Some indication of the order of magnitude of the electrostatic fields that would result from a departure from electrical neutrality over an appreciable volume may be obtained by considering a hydrogen isotope plasma in which the density of both ions and electrons is 10^{15} particles/cm^3. Suppose that in some manner all the electrons present in a sphere of plasma of r cm radius were suddenly removed; the strength of the resulting electrostatic field E, as derived from Gauss's law, would be

$$E = \frac{Q}{r^2}, \tag{4.23}$$

where Q is the value of the charge removed. If n_e is the electron number density of the plasma, then

$$Q = \tfrac{4}{3}\pi r^3 n_e e,$$

when e is the electronic charge. Upon substituting this result into equation (4.23), it follows that

$$E = \tfrac{4}{3}\pi r n_e e.$$

4.42. If the radius of the sphere is taken as 1 cm, then since n_e is 10^{15} electrons/cm^3 and e is 4.80×10^{-10} statcoulomb, it follows that

$$\begin{aligned} E &= \tfrac{4}{3}\pi \times 10^{15} \times 4.80 \times 10^{-10} \\ &= 2 \times 10^6 \text{ statvolts/cm} \\ &= 6 \times 10^8 \text{ volts/cm}, \end{aligned}$$

so that the field strength has an enormous value. A departure of only 1 part in a million from charge equality would give rise to a field of 600 volts/cm near a sphere of radius 1 cm. Since E increases in proportion to r, at a given plasma density, the field strength would increase with the radius of the sphere of plasma [7].

THE SHIELDING DISTANCE

4.43. Although on a macroscopic scale the distribution of positive and negative charges in a plasma must be the same, there are, in a sense, microscopic deviations from neutrality. Consider, for example, a small volume element in the vicinity of a positive ion. As a result of the thermal motion of the charged particles there will sometimes be an excess of positive charges and sometimes an excess of negative charges in this volume element. However, if a time average is taken, it will be found, as a consequence of the electrostatic field, that the negative charge density due to electrons will exceed the positive charge density of the ions. The reverse situation will of course exist in a volume element near to an electron. The difference between the positive and negative charge densities will obviously be greater in the immediate vicinity of any given charged particle and it will fall off with increasing distance.

4.44. The foregoing arguments lead to the conclusion that every charged particle may be regarded as being surrounded by an "atmosphere" having a net charge of opposite sign. The uniform distribution of these microscopic atmospheres throughout the plasma then leads to the macroscopic neutrality

described above. The effective radius of the oppositely charged atmosphere surrounding an ion can be obtained by a procedure analogous to that employed in the study of the solutions of electrolytes [13, 14]. By postulating that the charged particles in a field of varying potential energy have a Boltzmann distribution and assuming that the gradient of the electrical potential can be expressed, in the usual manner, by Poisson's equation, based on Coulomb's law, it can be shown that the radius λ_d of the atmosphere surrounding a positive ion is

$$\lambda_d = \left(\frac{kT}{4\pi n_e e^2}\right)^{1/2}, \tag{4.24}$$

where T is the kinetic temperature of both electrons and positive ions. If T is expressed in terms of kilo-electron volts (§2.13), this equation becomes

$$\lambda_d = 2.35 \times 10^4 \left(\frac{T}{n_e}\right)^{1/2} \text{ cm.} \tag{4.25}$$

4.45. The length λ_d, defined by equations (4.24) and (4.25), is called the *shielding* (or *screening*) *distance*.* It is a measure of the distance from an ion beyond which the atmosphere, with a net negative charge, screens off the Coulomb field of that ion from the field of another ion moving nearby. It is for this reason that the term "shielding distance" is used. The dependence of λ_d on the electron density, as derived from equation (4.25), is represented in Fig. 4.5 for the temperatures 1, 10, and 100 kev. It is evident that, for conditions of thermonuclear interest, e.g., an electron density of 10^{15} particles/cm^3 and a temperature of 100 kev, the shielding length is 7.5×10^{-3} cm. This is large compared with the distance between the ions and electrons at densities of interest, so that the number of electrons (and ions) included in the spherical volume having a radius equal to the shielding length, i.e., the shielding volume, is considerable. In the case under consideration, for example, it is $\tfrac{4}{3}\pi \times (7.5 \times 10^{-3})^3 \times 10^{15} = 1.8 \times 10^9$ particles of each sign, making a total of 3.6×10^9. The values for other conditions are shown in Fig. 4.6 [7].

4.46. Since the number of charged particles in the shielding volume is quite large, a particle can interact with many others in traversing a distance equal to the shielding distance. For this reason, as will be shown below, the effect of so-called long-range interactions is more important than that of short-range encounters in producing large-angle scattering of any given charged particle as it passes through the plasma. In treating the long-range encounters, the combined effect of all the particles within the shielding volume is regarded as a collective or statistical interaction.

4.47. Although it has no direct connection with the problem of collisions in a plasma, it may be mentioned here that the shielding distance is roughly equal

*It is often referred to as the Debye shielding distance (or length) since its derivation is based on the Debye-Hückel treatment of electrolytes.

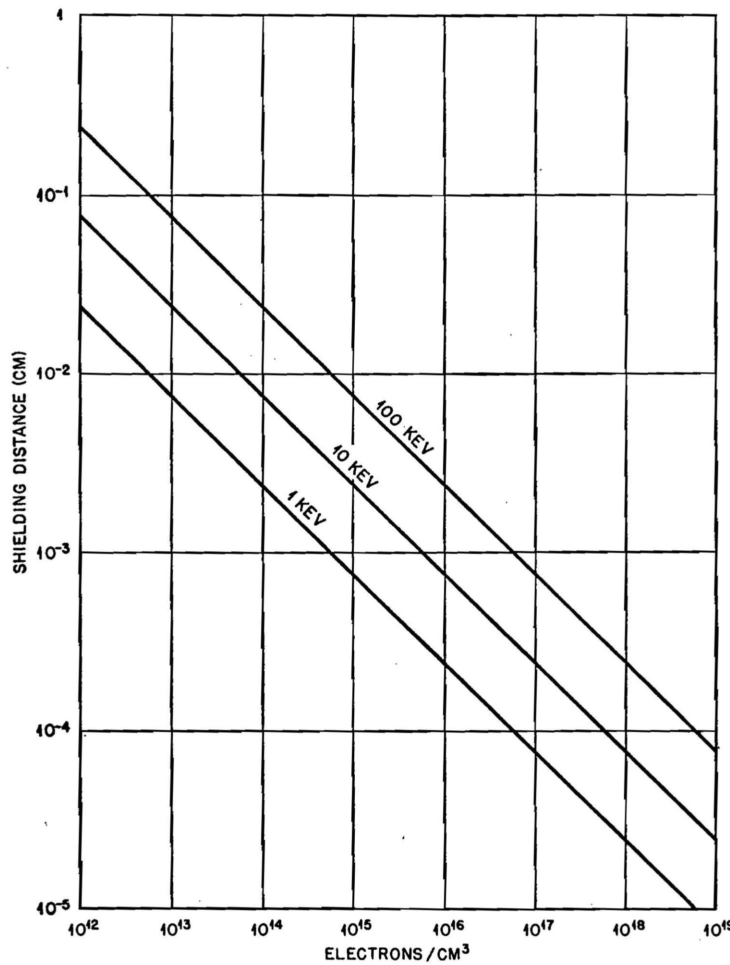

FIG. 4.5. Shielding distance at kinetic temperatures of 1, 10, and 100 kev.

to the thickness of the sheath which develops when a plasma is in contact with a solid surface (cf. §6.85). Since the electrons move faster than the ions, there will normally be a tendency for electrons to leave any given region of the plasma more rapidly than do the ions. At the floating potential of a solid surface in contact with a plasma, there is no net current flow to or from the surface, and so the rates at which positive and negative charges reach the

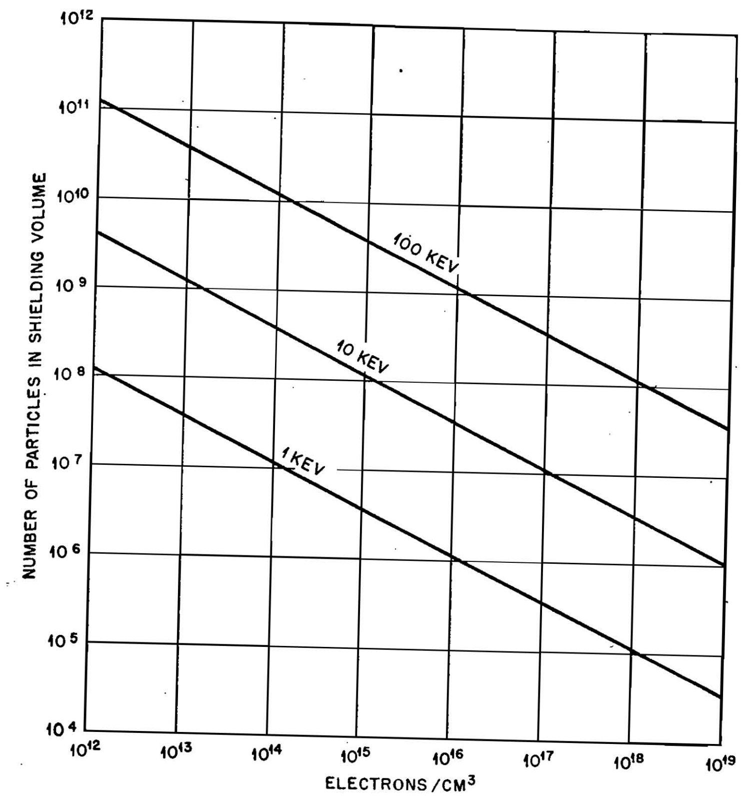

FIG. 4.6. Number of charged particles in shielding volume.

surface must be equal. This can be achieved only if there is a sheath of plasma near the solid surface where the number of electrons exceeds that of the ions. Within this sheath, which is somewhat analogous to the oppositely charged atmosphere surrounding a charged particle, electrical neutrality is not preserved.

SHORT-RANGE INTERACTIONS

4.48. An approximate value of the large-angle, single-collision cross section for short-range interaction (or close encounters) between charged particles may be obtained by a simple, classical treatment based on Coulomb's law. In the absence of any electrostatic forces, the distance of closest approach between two particles is called the *impact parameter*. The magnitude of this distance will determine the angle of deflection of one particle by the other, and for a deflection of 90°, let the impact parameter be b_0, as shown in Fig. 4.7. By making the simplifying assumption that the mass of the scattered particle is less than that of the scattering particle so that the latter remains essentially stationary during the encounter, it is found from Coulomb's law that, for a 90° deflection,

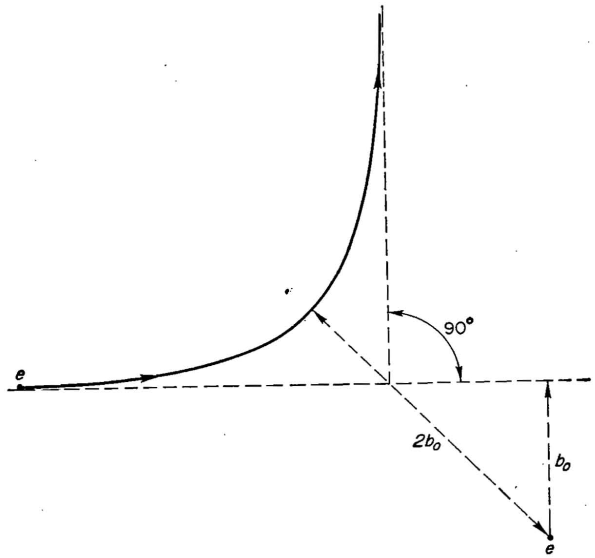

FIG. 4.7. Short-range Coulomb interaction for 90° deflection.

the particles are a distance $2b_0$ apart at the point of closest approach. At this point the mutual potential Coulomb energy is equal to the center-of-mass (or relative) kinetic energy W of the interacting particles. In the case of a hydrogen isotope plasma, all the particles carry the unit charge e, and the mutual potential energy at the point of closest approach is $e^2/2b_0$, and so

$$W = \frac{e^2}{2b_0}$$

or

$$b_0 = \frac{e^2}{2W}. \tag{4.26}$$

4.49. Such particles as approached each other with an impact parameter of b_0 or less will be scattered through large angles, i.e., 90° or more, in a single encounter. The cross section σ_c for close collisions of this type may be taken as roughly equal to the area of a disc with a radius b_0; hence,

$\frac{e^2}{m_0 c^2} = 2.818 \times 10^{-13}$ cm

$e^2 = (2.8 \times 10^{-13})$ cm $(.511\ \text{MeV})$

$e = 4.8 \times 10^{-10}$ esu

$$\sigma_c \approx \pi b_0^2 = \frac{\pi e^4}{4W^2}, \tag{4.27}$$

so that σ_c is inversely proportional to the square of the relative kinetic energy. The cross section will be given in square centimeters if e is expressed in statcoulombs and W in ergs. If W is in kilo-electron volts, the equation for the classical cross section for close collisions becomes

$$\sigma_c \approx \frac{1.6 \times 10^{-20}}{W^2}\ \text{cm}^2 = \frac{1.6 \times 10^4}{W^2}\ \text{barns}. \tag{4.28}$$

4.50. An interesting application of equation (4.28) is that in which accelerated deuterons bombard a cold target, since this is the basis of some methods that have been proposed for obtaining nuclear fusion energy (§3.53 *et seq.*). The short-range Coulomb interactions between hot deuterons and either cold deuterons or electrons of the target is then important. The cross sections for these processes can be derived from equation (4.28).

4.51. In general, for a collision between two particles of masses m_1 and m_2, the relative kinetic energy W is given by (cf. §2.4)

$$W = \frac{m_1 m_2}{2(m_1 + m_2)} v^2, \tag{4.29}$$

where v is the relative velocity. If particle 2 is essentially at rest, e.g., in a cold target, v is equal to v_1, the velocity of the hot (or accelerated) particle. For a deuteron-deuteron interaction, equation (4.29) reduces to

$$W = \tfrac{1}{4} m_D v_D^2 = \tfrac{1}{2} W_D,$$

where W_D is the kinetic energy of the deuteron. Hence, equation (4.28) for the close-collision cross section in this case is

$$\sigma_c \approx \frac{6.4 \times 10^4}{W_D^2}\ \text{barns}. \tag{4.30}$$

Thus, at an energy W_D of 100 kev, the value of σ_c is roughly 6.4 barns, i.e., about 200 times the total cross section for the D-D reactions at the same energy. For the bombardment of cold tritium by 100-kev deuterons, the close-collision cross section would be about 4.5 barns, which is not very different from that of the D-T fusion reaction.

4.52. For the collision of an accelerated ion with a cold electron in a target, $m_1 \gg m_2$, and equation (4.29) becomes

$$W \approx \tfrac{1}{2} m_e v_i^2$$

$$= \tfrac{1}{2} m_i v_i^2 \left(\frac{m_e}{m_i}\right) = W_i \frac{m_e}{m_i},$$

where m_i and m_e are the masses of the ion and electron, respectively, and W_i

is the energy of the accelerated ion. If this result is inserted into equation (4.28), it is seen that

$$\sigma_c \approx \frac{\pi e^4}{4W_i^2}\left(\frac{m_i}{m_e}\right)^2 = \frac{1.6 \times 10^4}{W_i^2}\left(\frac{m_i}{m_e}\right)^2 \text{ barns,} \tag{4.31}$$

with the ion energy W_i in kilo-electron volts. For deuterons of 100-kev energy, W_i is 100 and m_i/m_e is roughly 3660, so that the cross section for the short-range, large-angle (> 90°) scattering of cold electrons exceeds 10^7 barns. This is the value which was used in §3.54.

LONG-RANGE INTERACTIONS

4.53. The collision cross section derived above applies to a single Coulomb interaction that leads to scattering through an angle of 90° or more. As indicated in §4.39, however, consideration must also be given to the long-range, large-angle scattering (or distant collisions). These are the result of numerous small-angle Coulomb interactions, occurring at distances greater than the distance of closest approach defined by equation (4.26), that give a resultant large-angle scattering of about 90°. The effective cross section for this kind of scattering may be calculated in the following manner.

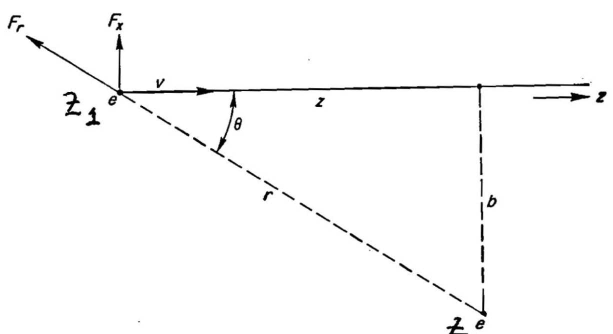

FIG. 4.8. Calculation of momentum transfer between charged particles.

4.54. Consider a (scattered) particle, with a charge e, moving in the z direction past another (scattering) particle having the same charge. Let r be the distance between the particles at any instant and b their impact parameter. The Coulomb force between the particles at the given instant is F_r and its component F_x at right angles to the z direction is equal to $F_r \sin\theta$, as may be seen from Fig. 4.8. The change of momentum Δp_x of the scattered particle in the x direction is then given by

$$\Delta p_x = \int_0^\infty F_x\,dt = \int_0^\infty F_r \sin\theta\,dt,$$

the integration being taken over all time. By Coulomb's law, $F_r = e^2/r^2$, and since $b/r = \sin\theta$, it follows that

$$\Delta p_x = \frac{e^2}{b^2}\int_0^\infty \sin^2\theta \sin\theta\,dt. \tag{4.32}$$

If v is the velocity of the scattered particle at the point under consideration, then $dz/dt = v$, so that $dt = dz/v$, and since $b/z = -\tan\theta$, it is readily found that

$$dt = \frac{b \operatorname{cosec}^2\theta\,d\theta}{v}.$$

Upon making this substitution into equation (4.32) and changing the limits, the result is

$$\Delta p_x = \frac{e^2}{bv}\int_0^\pi \sin\theta\,d\theta = \frac{2e^2}{bv}. \quad Z_1 Z_2 \tag{4.33}$$

This gives the change in momentum in the x direction resulting from a single scattering; the change is the same in the y direction.*

4.55. In the course of its motion the scattered particle experiences many deflections, most of them being small. The total momentum change Δp_x in a particular direction, e.g., along the x axis, is then given by

$$\Delta p_x = (\Delta p_x)_1 + (\Delta p_x)_2 + \cdots + (\Delta p_x)_j + \cdots + (\Delta p_x)_N, \tag{4.34}$$

where N is the total number of collisions made by the scattered particle with N scattering (or field) particles.† Since the successive small-angle collisions are assumed to be random in nature, it is impossible to predict the value of Δp_x. However, if many scattered particles are considered, each having the same initial velocity and direction of motion and making the same number of collisions, an average value, i.e., $\overline{\Delta p_x}$, can be determined. If the distribution of particle velocities in the plasma is isotropic, then $\overline{\Delta p_x}$ must vanish, from symmetry considerations, when all directions of motion are taken into account. However, $|\overline{\Delta p_x}|$ is not necessarily zero, and information concerning the change in momentum is best obtained by considering the mean square $\overline{(\Delta p_x)^2}$.

4.56. From equation (4.34), it is seen that

$$(\Delta p_x)^2 = (\Delta p_x)_1^2 + (\Delta p_x)_2^2 + \cdots + (\Delta p_x)_j^2 + \cdots + (\Delta p_x)_N^2 + (\Delta p_x)_1(\Delta p_x)_2 + \cdots + (\Delta p_x)_j(\Delta p_x)_k + \cdots \tag{4.35}$$

If $(\Delta p_x)_j$ in each collision is small, the average will be the same for all collisions; furthermore, taking into consideration the random nature of the deflections,

* It is of interest to note that the momentum change is equivalent to that resulting from the Coulomb force e^2/b^2, at the distance of the impact parameter b, operating over a path length of $2b$, i.e., for a time $2b/v$.

† The term "field particle" has been proposed because of its analogy with "field star," as used in astrophysics [15]. The scattered particle is sometimes called the "test particle."

the sum of the cross-product terms, such as $(\Delta p_x)_j(\Delta p_x)_k$, will vanish when averaged over all the particles. The same situation will apply, of course, to momentum changes along the y axis, and so equation (4.35) leads to the general result

$$\overline{(\Delta p)^2} = N\overline{(\Delta p)_j^2},$$

and upon differentiating it follows that

$$d\,\overline{(\Delta p)^2} = \overline{(\Delta p)_j^2}\,dN, \tag{4.36}$$

where $(\Delta p)_j^2$ may be taken as being equal to the square of Δp_x as given by equation (4.33).

4.57. If n is the number density of the scattering (or field) particles, the number dN of such particles contained in a cylindrical shell of length λ, radius b, and thickness db is

$$dN = 2\pi n\lambda b\,db.$$

Hence, equation (4.36) can be written as $z_1 z_2$

$$d\,(\Delta p)^2 = \left(\frac{2e^2}{bv}\right)^2 2\pi n\lambda b\,db$$

$$= \frac{8\pi e^4}{v^2}\,n\lambda\,\frac{db}{b}. \tag{4.37}$$

If this is integrated over all values of the impact parameter, from a minimum of $b_{\min}$ to a maximum of $b_{\max}$, the result is

$$\overline{(\Delta p)^2} = \frac{8\pi e^4}{v^2}\,n\lambda \ln \Lambda, \tag{4.38}$$

where Λ is defined by

$$\Lambda \equiv \frac{b_{\max}}{b_{\min}}. \tag{4.39}$$

4.58. Provided the mass of the scattered particle is about equal to or less than that of the scattering particle, e.g., in the scattering of electrons by electrons or by ions and of ions by ions, it can be assumed that when $\overline{(\Delta p)^2}$ has increased to the point at which it has roughly the same magnitude as p^2, where p is the initial momentum of the scattered particle, then the particle has been scattered through a large angle, e.g., about 90°.* The net distance λ traveled by the particle is formally equivalent to the mean free path for this scattering process, but it is also called the *relaxation length*, i.e., the distance over which the cumulative effect of many small deflections in long-range encounters is to produce a scattering angle of about 90° (see §4.39).

* This cannot apply to the scattering of ions by electrons because the ion, being considerably heavier than the electron, can lose much of its energy in encounters with electrons without suffering appreciable deflection.

It is then possible to define a cross section σ_d for the large-angle, distant scattering by

$$\lambda = \frac{1}{n\sigma_d}. \tag{4.40}$$

4.59. If m is the mass of the scattered particle, then $p = mv$, so that, when $\overline{(\Delta p)^2}$ is approximately equal to $(mv)^2$, where the scattered particle velocity v is now an average value, the length λ may be replaced by $1/n\sigma_d$. Hence, from equation (4.38), after replacing $n\lambda$ by $1/\sigma_d$, in accordance with equation (4.40),

$$(mv)^2 \approx \frac{1}{\sigma_d}\cdot\frac{8\pi e^4}{v^2}\ln\Lambda$$

or

$$\sigma_d \approx \frac{8\pi e^4}{m^2v^4}\ln\Lambda. \tag{4.41}$$

The kinetic energy W^* of the scattered particle is $\frac{1}{2}mv^2$, and so equation (4.41) reduces to

$$\sigma_d \approx \frac{2\pi e^4}{W^2}\ln\Lambda. \tag{4.42}$$

Upon comparison of this expression with equation (4.27), it is seen that, for equivalent energies, the cross section σ_c for close encounters leading to large-angle scattering is smaller than σ_d, the cross section for distant interactions, by a factor of 8 ln Λ.

4.60. In assigning values to $b_{\min}$ and $b_{\max}$, the ratio of which determines Λ, as defined by equation (4.39), it appears from the introductory discussion in §4.39 that $b_{\min}$ is equal to the impact parameter b_0 for a scattering of 90° in a single encounter, as defined by equation (4.26), and $b_{\max}$ is equal to the shielding distance λ_d, expressed by equation (4.24), beyond which the net effect of Coulomb forces becomes negligible. Hence, in accordance with equation (4.39),

$$\Lambda \approx \frac{\lambda_d}{b_0} = \left(\frac{kT}{4\pi n_e e^2}\right)^{1/2}\frac{2W}{e^2}.$$

If the energy distribution among the particles in the plasma is Maxwellian, W may be replaced by $\frac{3}{2}kT$, so that

$$\Lambda \approx \frac{3}{2e^3}\left(\frac{k^3T^3}{\pi n_e}\right)^{1/2} \tag{4.43}$$

$$= 4.9 \times 10^{14}\,\frac{T^{3/2}}{n_e^{1/2}},$$

with T expressed in kilo-electron volts and n_e in electrons/cm³.

* It should be noted that W is here the actual energy of the scattered particle, and not the relative energy as in equation (4.27).

4.61. Some values for $\ln \Lambda$ for plasma temperatures and electron densities in the range of thermonuclear interest, as calculated from equation (4.43), are given in Table 4.1.* It is seen that, under the probable conditions existing in a thermonuclear reactor, $\ln \Lambda$ would be in the vicinity of 20. Upon inserting this number into equation (4.41) and expressing W in kilo-electron volts, the result is

$$\sigma_d \approx \frac{2.6 \times 10^{-18}}{W^2}\ \text{cm}^2 = \frac{2.6 \times 10^6}{W^2}\ \text{barns}. \tag{4.44}$$

TABLE 4.1. APPROXIMATE VALUES OF $\ln \Lambda$

T (kev)	Electron Density n_e (particles/cm³)		
	10^{12}	10^{14}	10^{16}
0.1	16.5	14.2	11.9
1	20.0	17.7	15.4
10	23.4	21.1	18.8
100	26.9	24.6	22.3

[illegible] in plasmas ? validity of 4.44

4.62. Although it is obvious from equations (4.28) and (4.44) that the cross section σ_d for large-angle scattering as a result of distant collisions must exceed σ_c for close collisions, the exact ratio depends upon the actual nature of the particles involved, since W in equation (4.28) is the relative energy of the interacting particles, whereas in equation (4.44) it is the energy of the scattered particles. A situation of interest, as a possible means of achieving nuclear fusion (§3.63), is that involving the head-on collision of two beams of accelerated deuterons. The relative energy is then twice the energy of the scattered deuterons, so that σ_d is about 640 times σ_c. Since at 100-kev energy, for example, σ_c is more than 200 times as large as the total D-D reaction cross section (§4.51), it is apparent that σ_d will exceed the latter by a considerable factor. It is obvious, therefore, that when two beams of accelerated deuterons collide, Coulomb scattering will be far more probable than combination.

4.63. The expressions derived above are not exact, because of various approximations, including the neglect of quantum mechanical factors. Nevertheless, the results may be expected to hold, approximately, for encounters between particles of equal mass or where the scattered particle is lighter than the scattering particles. Thus, in a plasma produced from hydrogen isotopes, σ_d as derived from equation (4.41) gives a rough estimate of the effects of distant ion-ion, electron-electron, and electron-ion encounters. For the scattering of ions by electrons (cf. §4.58, footnote), i.e., ion-electron encounters, however, a different approach must be used, as described below.

* Because of quantum mechanical effects, these values must be corrected for temperatures in excess of about 40 ev, especially for electron-ion collisions. The changes are, however, not significant for the present purpose [16].

RELAXATION TIMES

4.64. The concept of collisional *relaxation times* cannot be very precisely defined;* it is nevertheless useful in the study of distant collisions for a known energy distribution. In general, the relaxation time is the time required for collisions to produce a large-angle scattering or a considerable change in the momentum (or energy) of a particle interacting with a background of other particles. Thus, it may be regarded as a measure of the rate of attempted approach of a nonequilibrium plasma distribution to an equilibrium state.

4.65. Three different cases are of special interest. The first is concerned with the time for a particular energetic ion or electron to be scattered in direction (or changed in energy) by encounters with field particles of the same kind. The second pertains to the effect of collisions of an electron with the ions as field particles, and the third applies to the reverse of this process, that is, the interaction of an ion with the background (or field) electrons. If λ is the relaxation length for a given interaction and v is the velocity of the scattered particle relative to the field particles, the average time elapsing before interaction occurs is λ/v or, since λ is equal to $1/n\sigma$,

$$t = \frac{1}{n\sigma v}, \tag{4.45}$$

where n is the number density of the field particles. Hence, the cross section σ_d for distant encounters derived above may be used to estimate relaxation times.

4.66. Although the individual field particles may have an arbitrary velocity distribution, the center of mass of a group of field particles is stationary. Hence, in determining the average value of the relaxation time, by means of the appropriate form of equation (4.45), v may be taken as the actual, rather than the relative, velocity of the scattered particle.

4.67. Using the subscripts e and i to indicate electrons and ions, respectively, it follows that t_{ii}, the mean collision time for an ion scattered by distant encounters with other ions, is thus

$$t_{ii} = \frac{1}{n_i \sigma_d v_i}, \tag{4.46}$$

where v_i is the average ion velocity. Similarly, for the scattering of electrons by electrons,

$$t_{ee} = \frac{1}{n_e \sigma_d v_e}, \tag{4.47}$$

* Relaxation times have long been used in astrophysics in the theoretical study of stellar encounters [2].