NPRE/ECE 421/ PHYS 479
2nd In Class Exam 4/13/2010
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All sub-section questions have equal weight.

1. Axial magnetic field gradients produce a force, $F_{/\!/}$, on charged particles that is proportional to the gradient. Consider a magnetic mirror configuration.
    a. Set up, starting from the force equation, the condition for "reflection" of the charged particle.
    b. Consider a 20 keV deuterium ion and a 20 keV electron trapped in the mirror field. What is the difference in axial position for reflection of these two particles?
    c. The force equation can be written in terms of the magnetic moment, u. This is considered to be a constant of motion if the field gradient is not "too large". Explain how to determine if the gradient is "too large" and why, physically, the magnetic moment then fails to be a constant of motion.

2. The fluid treatment of a plasma, combined with Maxwell's equations, yields the ideal MHD theory of a plasma.
    a. What is the major physical restriction (assumption) involved in treating the plasma as a fluid? If this breaks down, what theory should be used?
    b. What is assumed about plasma conductivity in ideal MHD theory? What is corresponding Ohm's law?
    c. The energy equation for the fluid plasma corresponds to the ideal gas, i.e., $pV = n\mathrm{RT}$. Justify this assumption, noting that differential form for conservation is
    $$\frac{dp}{dt} + \gamma p(\nabla \cdot \boldsymbol{v}) = 0$$
    d. Consider 20keV deuterium ions and electrons in the fluid plasma. How do the perpendicular diffusion coefficients, $D_{\perp}$, for the two compare?

3. Plasma stability theory for the fluid model can be derived using the ideal MHD theory.
    a. A "visual" stability requirement is that magnetic field lines are convex towards the plasma. Explain the physical interpretation of this requirement.
    b. Using sketches indentify all regions of bad curvature in a Tokomak; also for a mirror. Qualitatively, how are each stabilized against the bad curvature features?
    c. Fluid stability theory involves deriving the equation of motion, $F(\xi) = -\rho_{\circ} \frac{d^2\xi}{dt^2}$
    For the displacement vector ξ for a perturbed plasma. Explain the physical meaning of ξ. Based on this equation, what is the requirement for stability?
    d. The variation theory derived from $F(\xi)$ requires $\delta W > 0$. Where W represents work done by F. What is the physical interpretation of this stability requirement?
    e. The BBGKY kinetic theory for a plasma develops an equation for distribution function $f_N(\boldsymbol{r_1},\boldsymbol{v_1};\boldsymbol{r_2},\boldsymbol{v_2};\ldots;\boldsymbol{r_N},\boldsymbol{v_N};t)$. Explain why the BBGKY equations (e.g. 6.32) can not be solved without adding an assumed "closure equation". In the particles case, the closure equation uses in the homework was take as: $f_2 = f(1)f(2) + C(1,2)$. Explain the physical meaning of this assumption. How does it provide the required closure?

4. Performance of a fusion reactor can be analyzed by plotting its operating line vs. energy balance line on an $n\tau$-kT plot, giving approximate units on the axis.
   a. Indentify the following: region of excess power production, region of sub-ignition, desired point for operation.
   b. Assume your Tokomak intercepts the minimum ignition requirement. What change in its physics (including geometry) would be required to achieve a commercial “power plant”?
   c. Another company proposes building a stellarator to compete with your Tokomak reactor. They claim stability advantages, especially against complete (massive) plasma disruptions. Is their claim justified? Explain your answer.
   d. A so-called thermo instability can exist in a fusion reactor. Explain this with reference to your plot for question 4a. What can be done to prevent this?
   e. Your plot for question 4a. involves the energy confinement time while fueling requirements involve the particle confinement time. Explain the difference. What is the relationship between the two?
   f. Explain the difference between neoclassical and classical diffusion in a Tokomak. Which sets the theoretical upper limit on losses?