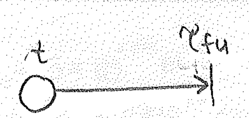
4.3



FOR STEADY STATE, τ_{fu} BECOMES

$$\tau_{fu,ss} \approx \frac{1}{\langle \sigma v \rangle_{dt} \, N_d}$$

$\Rightarrow$ $N_{d,0}$ now becomes N_d because the ions being consumed by ions but they are replaced by a Source (steady state)

The mean free path becomes

$$\lambda_{fu} = V_i \cdot \tau_{fu,ss}$$

The mean free path is the average distance that each ion travels from the time it is injected until it fusions.

An analogy to this can be seen by the following example:

The average distance a football travels from the beginning of a play till the end of the play in each down (American football).

$\boxed{4.4}$

$N_{i,0} R_b = 5\times10^{+28}\ m^{-2}$, $R_b = 0.25mm$

$\Rightarrow N_{i,0} = 2\times10^{23}\ mm^{-2}$

$N_d = N_t = \frac{1}{2} N_{i,0} = 1\times10^{23}\ \frac{reactions}{mm^3} \times 10\%\ burnup = 1\times10^{22}\ \frac{reactions}{mm^3}$

$1\times10^{22}\ \frac{reactions}{mm^3} \times \frac{4}{3}\pi(0.25mm)^3 = 6.545\times10^{20}\ \frac{reactions}{pellet}$ (ENERGY)

$6.545\times10^{20} \times 17.6\ \frac{MeV}{reaction} \times \frac{1\times10^{6}\ eV}{1\ MeV} \times \frac{1.60022\times10^{-19} J}{1} = 1.846\times10^{9}$ Joules

$P = 5.5\ GW = 5.5\times10^{9}\ \frac{J}{s}$

$t = \frac{1.846\times10^{9}}{5.5\times10^{9}} = 0.336$ seconds

$\Rightarrow$ INJECTION RATE : $\boxed{2.98\ \frac{PELLETS}{SEOND}}$

✓

$\boxed{4.5}$

$N_i = N_e = 10^{20}\ m^{-3}$

$KT_i = 0.9KT_e = 18keV$

$\beta_{max} = 8\%$

$$\beta_{max} \frac{B^2}{2\mu_0} \geq N_i K T_i + N_e K T_e$$

$\Rightarrow \mu_0 = 1.2566\times10^{-6}\ \frac{m\cdot kg}{S^2 A^2}$

B MIN :

$$B^2 = \frac{2\mu_0 (N_i K T_i + N_e K T_e)}{\beta max}$$

$KT_i = 18keV \cdot \frac{1000eV}{1keV} \cdot \frac{1.6022\times10^{-19} J}{1eV} = 2.884\times10^{-15}\ \frac{kg\cdot m^2}{s^2}$

$KT_e = 20keV \cdot \frac{1000eV}{1keV} \cdot \frac{1.6022\times10^{-19} J}{1\ eV} = 3.2044\times10^{-15}\ \frac{kg\cdot m^2}{s^2}$

$$B^2 = \frac{2(1.2566\times10^{-6}\ \frac{m\cdot kg}{s^2 A^2})\left[(10^{20}\ \frac{1}{m^3})(2.884\times10^{-5}\ \frac{kg\cdot m^2}{s^2}) + (10^{20})(3.2044\times10^{-15})\right]}{0.08}$$

$B^2 = 19.127$

$\boxed{B = 4.37\ T}$

✓

$\boxed{4}$

$n = n_o$

$f_b = \frac{n_o}{4} \langle \sigma v \rangle \tau$

FROM EQN 2.

$\tau = \frac{R_b}{V_{sound}} = R_b \left(\frac{3 \bar{m}_i}{10 k T_i} \right)^{1/2}$

5.5 GW_t

$f_b = \frac{\frac{n^2}{4} \langle \sigma v \rangle \tau}{n_o}$

- $\rho R = 3\ g/cm^2$ at $kT = 20 keV$
- $\rightarrow R = 3 \times 10^{-3}$ cm
- assume $n = n_o$
- $kT = 20$ OR 100

(1)

$\bar{m} = \frac{m_d + m_t}{2} = \frac{3.3446 \times 10^{-27} kg + 5.0084 \times 10^{-27} kg}{2} = 4.1765 \times 10^{-27}$ kg

$\tau = (3 \times 10^{-5} m) \left(\frac{(3) 4.1765 \times 10^{-27} kg}{10\,(20\,keV)\left(\frac{1000\,eV}{1\,keV}\right)\left(\frac{1.6022 \times 10^{-19} J}{1 eV}\right)} \right)^{1/2} = 1.876 \times 10^{-11}$ sec

$\rho R = 3 \frac{g}{cm^2} \Rightarrow \rho = 1000 \frac{g}{cm^3} \left(\frac{1}{4.1765 \times 10^{-27} kg} \times \frac{1 kg}{1000 g} \right)$

$n = 2.394 \times 10^{26} \frac{ions}{cm^3} = 2.394 \times 10^{32} \frac{ions}{m^3}$

$f_b = \frac{2.394 \times 10^{32}}{4\ m^3} \langle 4.31 \times 10^{-22} \frac{m^3}{s} \rangle (1.876 \times 10^{-11} sec)$

$\boxed{f_b = 0.484}$

(2)

EQN 1: IS THE AVERAGE TIME BETWEEN FUSION EVENTS

$\frac{N_i}{2} = N_D \Rightarrow \frac{dn}{dt} = -\frac{n^2}{4} \langle \sigma v \rangle = -\frac{n}{\tau} \Rightarrow \tau = \frac{4 \langle \sigma v \rangle}{n}$

$RR = \left(\frac{N_i}{2}\right)^2 \langle \sigma v \rangle$

EQN 2: Time for doubling of the pellet radius. From it's initial radius R_b to get to $2R_b$. In this time the rate of fusion energy release decreases by a big amount. So most fusion reactions will take place in this time

$\boxed{5}$ a) $I^2 = 200\ NkT$

(1) $p = \frac{B_\theta^2}{8\pi}$

$B_\theta = \frac{\mu_0 I}{2\pi r} = 2\times10^{-7}\frac{I}{r}$

(2) B_θ(gauss) $= \frac{I\,(amp)}{5r\,(cm)}$

COMBINING (1) & (2)

$p = \frac{I^2}{200\pi r^2}$

$\Rightarrow I^2 = 200NkT$ (kT in ergs)
(where $N = \pi r^2 n$)

b) FOR $\beta < 1$

$$\beta_{max}\ \frac{B^2}{2\mu_0} \geq N_i k T_i + N_e k T_e$$

$p = \frac{B_\theta^2}{8\pi} - \frac{B^2}{8\pi}$

$p = \frac{B_\theta^2}{8\pi}\left(1 - \frac{B^2}{B_\theta^2}\right) = \frac{B_\theta^2}{8\pi}\beta$ Let $\beta = \left(1 - \frac{B^2}{B_\theta^2}\right)$

$B_\theta = \frac{\mu_0 I}{2\pi r} \Rightarrow B_\theta$(gauss) $= \frac{I\,(amp)}{5r\,(cm)}$

$p = \left(\frac{I^2}{200\pi r^2}\right)\beta$

$I^2 = \frac{200\ NkT}{\beta}$