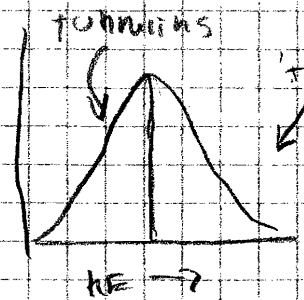
NPRE 241 Tuesday February 2nd.

26 → its ok to just get equation,

Last time



Astrophysical Fusion

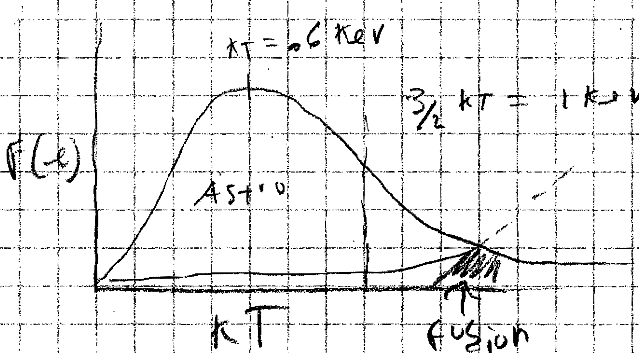
~ keV range

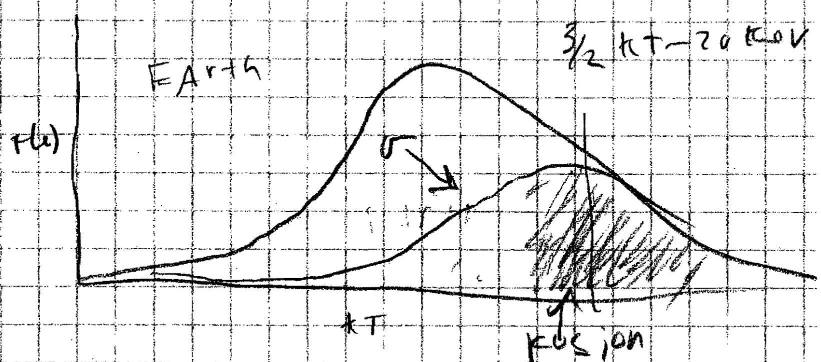
terrestrial Fusion

> 20 keV

$$\boxed{\begin{aligned} 2\pi\eta &= .9895\, Z' Z^2 \left(\frac{A}{E}\right)^{1/2} \\ \sigma(E) &= S(E)\,\frac{\exp(2\pi\eta(E))}{E} \end{aligned}} \approx \frac{1}{v_r}\exp\left(-\gamma\frac{q_1 q_2}{v_r}\right)$$

$S(E)$ = Astrophysical S-factor.





Distribution functions

$F(E)$: $\frac{\#\text{ of particles}}{cm^3 \cdot \text{unit } E}$

$$n = \int_0^\infty F(E)\, dE$$

pos, vel, time

$f(\bar{r}, \bar{v}, t)$: # at $\bar{r}$ with $\bar{v}$ at t

$$F(\bar{r}, t) = \int f(\bar{r}, \bar{v}, t)\, d^3v$$

$$f(\bar{r}, v, t) = \int f(\bar{r}, \bar{v}, t)\, d\Omega$$

$F(E)\,dE$, want $F(v)\,dv \big|_{v=E}$ → use $E = \frac{1}{2}mv^2$

Form Jacobian

$$F(E) = \left| F(v) \frac{dv}{dE} \Big|_{E=\frac{1}{2}mv^2} \right|$$

$$\overline{E} = \int_0^{\infty} F(E)\,dE$$

$$\langle E \rangle = \frac{dF(E)}{dE}\Big|_{E=0}$$

kT = kinetic temp of plasma
$\frac{3}{2}kT$ = Average E of Maxwellian plasma

$\frac{1}{2}kT$ = most frequent particle energy in max. plasma

$\left(\frac{8kT}{m\pi}\right)^{1/2}$ = Average particle speed.

$$RR \sim n_a n_b |\bar{v}_a - \bar{v}_b| \sigma(\bar{v}_a - \bar{v}_b)$$

↑ All speeds equal

Def: thermonuclear fusion: implies equilibrium which implies Maxwellian distribution

if thermonuclear fusion then you can average over $F(E)$
if you don't, you can't

not all speeds equal

$$\boxed{RR \sim \int n_a n_b \underbrace{|\bar{v}_a - \bar{v}_b| \sigma(\bar{v}_a - \bar{v}_b) f_a(\bar{v}_a) f_b(\bar{v}_b)}_{} d^3v_a\, d^3v_b}$$

Average $\sigma v_{ab} = \langle \sigma v \rangle_{ab}$

$RR \sim n_a n_b \langle \sigma v \rangle_{ab}$ $\qquad E = RR \cdot E_{fusion}$

units: $\frac{\#}{cm^3} \cdot \frac{\#}{cm^3} \cdot \frac{\cancel{cm^2} \cdot \cancel{cm}}{s} \cdot E \quad \frac{\#}{cm^3 s} \cdot E = \frac{W}{cm^3}$

$\langle \sigma v \rangle_{ab} \approx 10^{-21} \frac{m^3}{s} = \approx 80\ keV$

example (magnetic)

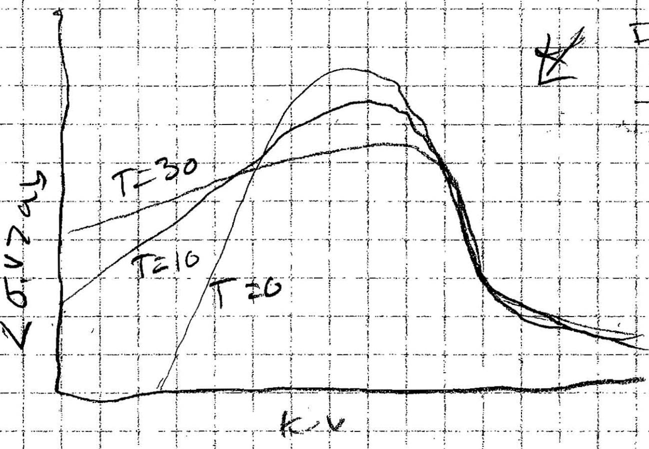
$a = D \qquad b = T$

$n_D = n_T = 10^{14}\ cm^{-3}$, $\langle \sigma v \rangle_{ab}\ 10^{-15} \frac{cm^3}{s}$

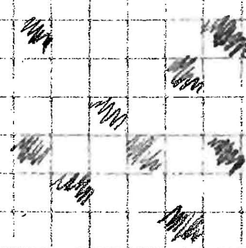
$n_D n_T \langle \sigma v \rangle_{ab} = 10^{15}$ rxn per second

$E_{fusion} = 20 \times 10^6\ eV \rightarrow E = 10^{15} \cdot 20 \times 10^6 = 20 \times 10^{14} \frac{ev}{cm^3 s} = \boxed{30 \frac{W}{cm^3}}$

if distribution ~~than~~ is non maxwellian then you must change $F_a(v_a)$ and $F_b(v_b)$ and redo integrals to get a new $\langle \sigma v \rangle_{ab}$



Injected D beam into T ions



electrons

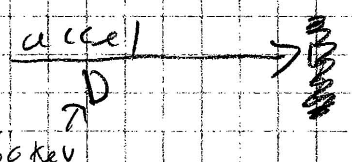
$\left.\begin{matrix} T_i : T_D, T_T \\ T_e \end{matrix}\right\}$ different equilibrium temperatures.

unless you inject ions you can assume ion temperatures are approximately equal

if you have a heated plasma $T_e \neq T_i$

but we don't care about T_e, so it's all good.

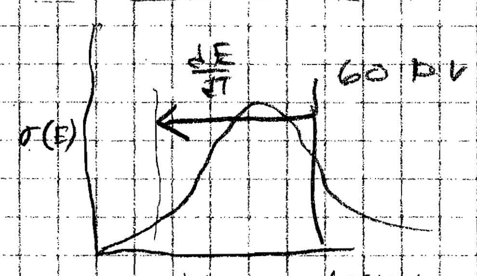
thought expiriment



T in Pd $kT_T \approx 0$, not a plasma.

E out << Ein

$n_D E_D \cdot v$ $\quad$ $n_D n_T \langle \sigma v \rangle_{DT}$



kT, slows down too rapidly for fusion to occur.

Conclusion: heat electrons up to decrease $\frac{dE}{dt}$ loss to increase time available ~~to~~ for fusion to occur.