effects on the positive and negative particles. Furthermore, according to Earnshaw's theorem of classical electrostatics, there is no position of stable equilibrium for a charged particle in an electrostatic field, no matter how complex its structure [1]. A consequence of this theorem is that no configuration of charges, such as a plasma, can exist in stable equilibrium under the influence of purely electrostatic forces [4].

3.13. In addition to the foregoing qualitative arguments, there is a quantitative reason why confinement of a plasma by means of an electrostatic field is not likely to be practical. An essentially static electric field can exert an effective pressure on a system of charged particles which is limited to the energy density of the field, given by $E^2/8\pi$.* If E is the field strength in statvolts/cm, then the energy density (or pressure) will be obtained in ergs/cm³ (or dynes/cm²). The pressure of the plasma, treated as an ideal gas, is nkT, where n is the total number of particles per cubic centimeter, i.e., the total particle density of the plasma. If the number densities of ions and electrons are each 10^{15} particles/cm³, then n is 2×10^{15} particles/cm³; and suppose T (or rather kT) is 100 kev. The minimum value of E required to contain the plasma is found by equating $E^2/8\pi$ to nkT; thus,

$$\frac{E^2}{8\pi} \geqslant 2 \times 10^{15} \times 100 \times 1.6 \times 10^{-9} = 3.2 \times 10^{8} \text{ ergs/cm}^3,$$

where 1.6×10^{-9} is the factor for converting kilo-electron volts into ergs. It is seen that E must be nearly 9×10^4 statvolts/cm or about 2.7×10^7 volts/cm. Thus, a stationary electrostatic field of impossibly large magnitude would be required to confine a plasma of reasonable particle density such as might be used in a thermonuclear reactor.

CONFINEMENT BY MAGNETIC FIELD

3.14. It will be shown in Chapter 4 that in a magnetic field charged particles gyrate about the lines of force, the positive particles in one direction and the negative particles in the opposite direction. Hence, apart from the effect of collisions, in a uniform magnetic field the ions and electrons remain tied to the field lines. Although they can move freely along (or parallel) to these lines, in either direction, they cannot cross the lines if there are no collisions among the particles. Hence, if the ions and electrons in a plasma can in some manner be prevented from escaping at the ends of the containing vessel, e.g., by means of an endless tube of toroidal form or in other ways, the use of a magnetic field appears to offer promise for confinement of a plasma. It remains to be seen, however, if the strength of the field required would be reasonable. For this purpose it is necessary to determine the relationship between the field strength and the effective pressure which the field could exert on the plasma. A simplified derivation of this relationship is given below.

* The equivalent expression for a magnetic field is derived below (§3.20).

3.15. The plasma will be regarded as consisting of singly charged positive and negative particles, i.e., hydrogen isotope nuclei and electrons, moving independently, so that collision forces can be neglected. If, in general, both electric and magnetic fields are present, the force in dynes acting on a single particle of charge $\pm e$ statcoulombs due to the electric field $\mathbf{E}$ statvolts/cm is $\pm eE$; and that due to the magnetic field of $\mathbf{B}$ gauss is $e(\mathbf{v} \times \mathbf{B})/c$, where $\mathbf{v}$ is the particle velocity in cm/sec and c is the velocity of light in the same units.

3.16. In the steady state, the force on all the particles in unit volume, i.e., the force density, is just balanced by the rate of momentum transfer which is equal to the gradient of the pressure. It follows, therefore, that for a plasma containing n_i ions/cm³ having an average velocity $\mathbf{v}_i$,

$$n_i e\left[\mathbf{E} + \frac{1}{c}(\mathbf{v}_i \times \mathbf{B})\right] = \nabla p_i \qquad (3.9)$$

and for the n_e electrons/cm³ with an average velocity $\mathbf{v}_e$,

$$-n_e e\left[\mathbf{E} + \frac{1}{c}(\mathbf{v}_e \times \mathbf{B})\right] = \nabla p_e \qquad (3.10)$$

where ∇p_i and ∇p_e represent the pressure gradients due to ions and electrons, respectively. In a hydrogen isotope plasma, n_i and n_e are equal, and so addition of equations (3.9) and (3.10) gives

$$\frac{ne}{c}\left[(\mathbf{v}_i - \mathbf{v}_e) \times \mathbf{B}\right] = \nabla p, \qquad (3.11)$$

where n is equal to both n_e and n_i, and ∇p is the total pressure gradient in the plasma. The quantity $ne\,(\mathbf{v}_i - \mathbf{v}_e)$ is the net rate of movement of charge and so is equal to the current density $\mathbf{j}$; it follows, therefore, from equation (3.11) that

$$\frac{1}{c}(\mathbf{j} \times \mathbf{B}) = \nabla p. \qquad (3.12)$$

3.17. For an electric field which does not vary with time, Maxwell's equation (3.8) reduces to

$$\nabla \times \mathbf{B} = \frac{1}{c}(4\pi \mathbf{j}), \qquad (3.13)$$

and combination with equation (3.12) gives

$$\frac{1}{4\pi}(\nabla \times \mathbf{B}) \times \mathbf{B} = \nabla p. \qquad (3.14)$$

In general,

$$(\nabla \times \mathbf{B}) \times \mathbf{B} = -\tfrac{1}{2}\nabla B^2 + (\mathbf{B}\cdot\nabla)\mathbf{B},$$

and in a magnetic field in which the lines of force are straight and parlel, the last term on the right is zero; upon making this substitution and rearranging, equation (3.14) becomes

$$\nabla \left(p + \frac{B^2}{8\pi}\right) = 0,$$

that

$$p + \frac{B^2}{8\pi} = \text{constant.} \tag{3.15}$$

3.18. The quantity $B^2/8\pi$, which has the units of energy/volume, is the *ergy density* of the magnetic field. However, since energy/volume is equivant to force/area, $B^2/8\pi$ is also regarded as the *magnetic pressure* of the field. follows, therefore, from equation (3.15) that the sum of the kinetic and agnetic pressures is constant in a plasma in a straight magnetic field. If ne plasma is completely confined by an external magnetic field of strength B_0, the pressure at the outside must fall to zero, and so it is seen that the onstant in equation (3.15) is equal to $B_0^2/8\pi$; that is,

$$p + \frac{B^2}{8\pi} = \frac{B_0^2}{8\pi}. \tag{3.16}$$

Hence, an external magnetic field B_0 can confine a plasma having a kinetic pressure p and a contained field B, where B_0, B, and p are related by equation (3.16). It will be noted that in the steady state the magnetic field within a plasma having a finite kinetic pressure must always be less than the external field. Consequently, a plasma confined by a magnetic field tends to be diamagnetic.

3.19. In the foregoing treatment, ∇p has been treated as the gradient of an isotropic scalar plasma pressure. A more rigorous derivation of equation (3.12) shows, however, that ∇p is really $\nabla \cdot \mathbf{p}$, the divergence of a stress tensor [5].* The pressure (or stress) tensor may then be described in terms of two equal scalar components $p_\perp$ at right angles to the field lines and the component $p_\parallel$ parallel to these lines. The p term in equation (3.16) is then strictly $p_\perp$; this means that the magnetic field can support a plasma (scalar) pressure only in the direction perpendicular to the field lines. Since the plasma exerts its pressure in all directions, escape of the particles along the field lines is possible, as indicated above, unless steps are taken to prevent it.

3.20. It will be seen in subsequent chapters that it is often convenient to express the kinetic pressure of the particles in a plasma in terms of its ratio to the external magnetic pressure (or energy density). The dimensionless ratio β is then defined by†

* In addition, the term $\rho \mathbf{v} \nabla \cdot \mathbf{v}$ should appear on the right of equation (3.12), but this is usually taken to be small in a quiescent plasma (cf. §13.47).

† A few writers have used the symbol β to represent the ratio of the kinetic pressure at a given point to the magnetic energy density *at the same point*.

$$\beta \equiv \frac{p}{B_0^2/8\pi}, \tag{3.17}$$

so that equation (3.16) may be written as

$$\beta = 1 - \frac{B^2}{B_0^2}. \tag{3.18}$$

Since the minimum value of B is zero, the ratio β has a maximum value of unity; this would represent the ideal case of a perfectly diamagnetic plasma from which the magnetic field was completely excluded. In this event, equation (3.16) may be written as

$$p_{\text{max}} = \frac{B_0^2}{8\pi}, \tag{3.19}$$

where p_{max} is the maximum kinetic pressure of a plasma that can be confined, in a steady state, by an external magnetic field of strength B_0.

3.21. Although stability requirements frequently restrict β in a magnetically confined plasma to values appreciably less than unity, equation (3.19) may be used to indicate the order of magnitude of the field that would be required to confine a plasma under conceivable conditions in a thermonuclear reactor. As was seen above, the kinetic pressure may be set equal to nkT, where n is the total particle density. Taking n as 2×10^{15} particles/cm³ and the temperature (or kT) as 100 kev, then

$$\frac{B_0^2}{8\pi} \approx 1.6 \times 10^8 \text{ ergs/cm}^3 \text{ (or dynes/cm}^2\text{)}$$

and, consequently,

$$B_0 \approx \sout{9 \times 10^4}\ 6.3 \times 10^4 \text{ gauss.}$$

An external field of about 90,000 gauss would thus be required to confine the specified plasma. A field of this order is by no means outside the realm of practicality, provided the dimensions of the containing vessel are not too large. It is seen, therefore, that confinement of the plasma in a thermonuclear reactor by means of a magnetic field should be a definite possibility. Attention may be called to the fact that the strength of available magnetic fields sets an upper limit of 10^{15} or 10^{16} particles/cm³ for the plasma density. It is a fortunate circumstance that this corresponds to a situation in which the power density has a reasonable value (§2.42).

3.22. It will be seen in the next chapter (§4.14) that the radius of gyration r_g cm of a charged particle around the lines of force of a magnetic field, in a hydrogen isotope plasma, is given by

$$r_g = \frac{mv_\perp c}{eB}, \tag{3.20}$$

where m grams is the mass of the particle, $v_\perp$ cm/sec is its velocity of gyration

·pendicular to the field direction, B gauss is the magnetic field strength, e tcoulombs is the electronic charge, and c cm/sec is the velocity of light. ıce there are two degrees of freedom perpendicular to the field lines, the ergy of gyration, $\frac{1}{2}mv_{\perp}^2$, is equal to kT for a Maxwellian distribution; ıce, equation (3.20) may be written as

$$r_g = \frac{c(2mkT)^{1/2}}{eB}. \qquad (3.21)$$

a temperature of 100 kev, i.e., $kT = 100$ kev $= 1.6 \times 10^{-7}$ erg, and a field ·ength of 100,000 gauss, the gyration radius would be 0.010 cm for an electron, ;0 cm for a deuteron, and 0.72 cm for a triton.*

3.23. As a result of inhomogeneities in the magnetic field, of collisions ıong the particles, and of the effects of electric fields, the particles in a plasma ll not rigorously follow the magnetic lines of force, but will tend to drift ross them to some extent. Nevertheless, to a first approximation, the radii gyration calculated above indicate the distances from a particular line of rce within which the respective charged particles remain when confined by magnetic field. Assuming that the lines of force do not intersect the material walls of the vessel containing the plasma, so that the particles are not :tually led into them, it seems that in a vessel of reasonable dimensions, e.g., , least several centimeters, the number of charged particles reaching the walls, ; a result of gyration about the field lines, will be relatively small. At lower ·mperatures and higher field strengths, the values of r_g are decreased, and so .so will be the wall losses from the source under consideration.

3.24. A number of methods for utilizing a magnetic field to confine a plasma ; high temperatures have been proposed. Four of these in particular have een the subject of experimental study, and their basic principles will be outned here. Further details, as well as a discussion of methods of producing and eating the plasma, will be given in later chapters. At the present time it is npossible to say which, if any, of the confinement methods to be described ·ill prove successful. One of the main difficulties is concerned with the fact ıat, except in special circumstances, a plasma confined by a magnetic field ; subject to instabilities which can result in the energetic particles reaching he walls of the containing vessel. As was seen above, this would lead to a eneral lowering of the temperature of the reacting system. The problem, ·hich is as yet far from being solved, is to overcome the instabilities for a ufficient time for the temperature of the plasma to be raised above the thermoıuclear ignition temperature, and also to satisfy the requirement that the ›roduct of the particle density and the reaction time exceed a certain minimum ·alue.

* For a particle of charge Ze and mass m, the value of r_g is proportional to $m^{1/2}/Z$; ıence the gyration radius of a He^4 nucleus, which is one of the products of the D-T ·eaction, will be less than that of a deuteron. The value for a He^3 nucleus will be smaller ;till.

METHODS OF PLASMA CONFINEMENT *

THE PINCH EFFECT

3.25. One method of confinement, which was proposed independently by a number of investigators in various parts of the world, makes use of what is called the *pinch effect;* it is the self-constriction that occurs in a plasma as a result of the passage of a unidirectional current (see Chapter 7). Such a current produces an azimuthal self-magnetic field (Fig. 3.1) that tends to

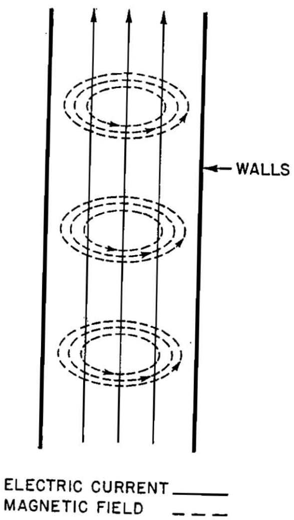

FIG. 3.1. Confinement of plasma by azimuthal self-magnetic field.

constrict (or pinch) the plasma. The phenomena is equivalent to the familiar one in which parallel circuits carrying current in the same direction attract each other.

3.26. If the discharge current is large enough, the constricting effect of the self-magnetic field can pull the plasma away from the walls of the containing vessel, so that magnetic confinement, in the sense discussed earlier, can be achieved. The fact that the discharge becomes pinched is direct evidence that the current carriers, i.e., the charged particles, are being prevented by the azimuthal self-magnetic field from moving in a radial direction.

3.27. An expression for the magnitude of the plasma current required to produce a pinched discharge will be derived in Chapter 7, but its physical

* The material in this section is intended to be introductory only, and so appropriate references are deferred to later chapters where the topics are discussed in detail.

basis will be apparent from the following simplified treatment. If there is no magnetic field trapped within the plasma in a pinched discharge, equation (3.19) takes the form

$$p = \frac{B_\theta^2}{8\pi}, \tag{3.22}$$

where the kinetic pressure p of the particles in the plasma is balanced, in the steady state, by the pressure (or energy density) of the azimuthal self-magnetic field B_θ produced by the discharge. Actually, the field strength decreases with distance from the discharge, and the value considered here is that just outside the pinched plasma.

3.28. To express the relationship between the discharge current and the magnetic field, mks units are used; thus, according to the Biot-Savart law, a current of I amp flowing in a conductor, e.g., the plasma, of radius r meters produces an azimuthal magnetic field B_θ webers/meter^2 given by

$$B_\theta = \frac{\mu_0 I}{2\pi r} = 2 \times 10^{-7}\frac{I}{r}, \tag{3.23}$$

since $\mu_0/4\pi$ is equal to 10^{-7}. In pinch studies, B_θ is generally stated in gauss, i.e., 10^{-4} webers/meter^2, and r in cm, so that

$$B_\theta\ (\text{gauss}) = \frac{I\ (\text{amp})}{5r\ (\text{cm})}, \tag{3.24}$$

and combination with equation (3.22) gives

$$p = \frac{I^2}{200\pi r^2}.$$

Expressing the kinetic pressure as nkT, with kT in ergs, it follows that

$$I^2 = 200NkT, \tag{3.25}$$

where N, equal to $\pi r^2 n$, is the linear particle density, i.e., the total number of particles per cm length of the discharge.

3.29. Some indication of the current that might be required to raise the temperature of a pinched plasma may be obtained from equation (3.25). Suppose the total particle density, i.e., hydrogen isotope nuclei and electrons, is 2×10^{15} particles/cm^3 and the cross-sectional area of the containing vessel is 1000 cm^2, so that N is 2×10^{18} particles/cm. If the temperature in the discharge is to reach 100 kev, the required current in amperes is given by equation (3.25) as

$$I^2 = 200 \times 2 \times 10^{18} \times 100 \times 1.60 \times 10^{-9}$$
$$= 64 \times 10^{12}$$

so that

$$I = 8 \times 10^6 \text{ amp}.$$

Thus, currents of the order of millions of amperes would be required to produce a pinched discharge under the conditions specified. For lower temperatures and particle densities, smaller currents would of course be adequate.

3.30. A pinched discharge can be most easily produced by applying a high voltage between two electrodes placed at the ends of a straight tube containing a gas at low pressure. Although linear discharges of this type have been commonly employed in the study of pinch phenomena, there might be two drawbacks in a thermonuclear reactor operating at very high temperatures. In the first place, since the electrodes are in contact with the plasma at all times, they may remove energy from the charged particles and thus lower the temperature. Furthermore, the sputtering from the electrodes could introduce elements of high atomic number into the plasma and so increase the energy lost as bremsstrahlung. Possibly because of the lower temperatures involved, neither of these effects has hitherto been serious in experimental work on linear pinched discharges.

3.31. The problems arising from the presence of electrodes can be overcome by the use of a toroidal tube to contain the gas. The latter acts as the secondary of a transformer in which an electric field is induced from an external primary circuit that partly or wholly surrounds the torus. The discharge within the gas then flows in a closed loop and no electrodes are needed. With a sufficiently large induced current in the plasma, the discharge is constricted, and can pull away from the walls, just as in a straight tube with electrodes within the gas.

3.32. One consequence of the use of an induced discharge is that the current flow cannot be steady. However, the intermittent (or pulsed) nature of the discharge has the advantage that, if rapid thermonuclear reaction can be achieved, some of the energy produced can be converted directly into electrical power. As the kinetic pressure of the plasma particles increases owing to the increase of temperature resulting from the release of nuclear fusion energy, the pinch can no longer be sustained; the constricted plasma will start to expand. The expanding plasma behaves like a mechanically driven armature in a conventional electric generator; in doing work against an external stationary magnetic field an emf is produced.

3.33. The outstanding difficulty in utilizing the pinch effect as a means for plasma confinement lies in the instability of the discharge. Several types of instability have been observed, and to these various names have been applied. There are, for example, the "sausage type" instability, or "necking off," in which the plasma becomes highly constricted at certain points so that it tends to break up into sausagelike links, and "kink" instability, and "wriggling," which are self-descriptive terms.

3.34. It appears from both theoretical arguments and experimental observations that the period of duration of the pinch can be extended, i.e., somewhat increased stability can be achieved, by (a) making the torus walls of a conducting material (or with a conducting coating); and (*b*) by including an axial (or longitudinal) magnetic field in the pinched discharge. This longitudinal field, with lines parallel to the direction of current flow in the plasma, as distinct from the azimuthal self-magnetic field, is supplied by a solenoidal winding around the torus. The trapping of the longitudinal magnetic lines of force within the constricted plasma adds to the stability of the latter. However, even with these modifications, the pinched discharge has been found to have a relatively short life, and various other developments are being considered, as will be explained in Chapter 7.

THE STELLARATOR

3.35. In principle, it should be possible to confine a plasma by means of an external magnetic field of sufficient strength produced by means of a solenoid. The situation in the case of a cylindrical tube is shown in Fig. 3.2, the magnetic

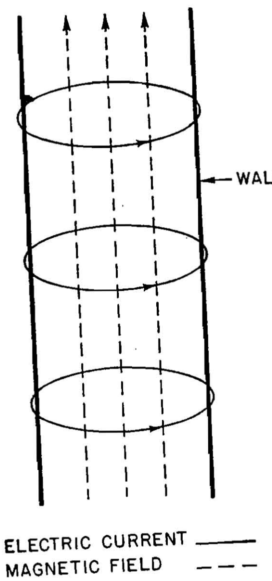

FIG. 3.2. Confinement of plasma by axial magnetic field.

lines of force being parallel to the cylinder axis. The minimum strength of the magnetic field required at the walls is given by equation (3.19) for the necessary particle density and temperature. Since the lines of force lead into the ends of the tube, the particles would lose considerable amounts of energy, and so the use of a torus in this case is again indicated. However, a fundamental difficulty now arises: the strength of the magnetic field in the plasma due to the current in the external solenoid decreases from the inner to the outer major radius of the torus. It will be shown in §4.36 that, in a nonuniform (or

inhomogeneous) magnetic field, charged particles tend to drift perpendicularly to the field lines, the electrons in one direction and the ions in the opposite direction. The resulting separation of charges produces an electric field which, in conjunction with the magnetic field, forces the plasma toward the walls. The accompanying energy loss will then be very large.

3.36. A partial solution to this difficulty is to twist the torus into a shape like a figure eight (Fig. 3.3), since the particles would tend to drift in opposite

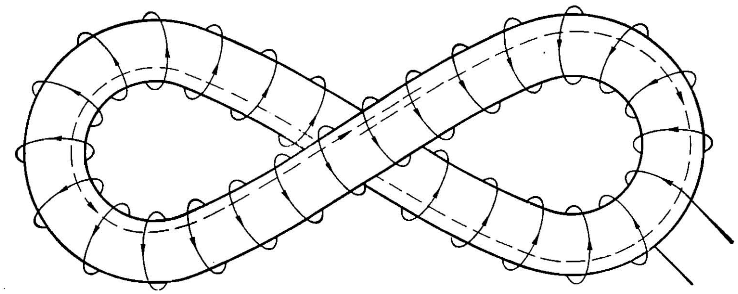

FIG. 3.3. Figure-eight form of stellarator.

directions in the two bends. This was the original form of the device called a *stellarator* (see Chapter 8), with the confining (axial) magnetic field produced by a current passing through a solenoid wound around the tube.

3.37. Basically, the difference in behavior of a plasma confined by an axial magnetic field in a planar torus and in one bent into a figure eight lies in a characteristic of the lines of force. If a line of force is followed around a planar torus, it should close upon itself. In the figure-eight configuration, however, this is not the case; after making a single circuit, the line of force is somewhat displaced from its original position. As a consequence of this displacement, a path becomes available for the charged particles to move upward or downward without crossing the lines of force. Hence, the charge separation which tends to occur, due to the variation in the magnetic field strength across the tube, can be partly neutralized. The drift of the plasma to the walls can thus be decreased.

3.38. The same result can be achieved, in principle, by any distortion of a planar torus or even in a planar torus if the lines of force are twisted by means of a helical magnetic field superimposed on the confining field. The name stellarator has been applied to systems in which the lines of force of an axial magnetic field are displaced in each circuit of a toroidal tube, by means of one of the methods mentioned above. ROTATIONAL TRANSFORM

3.39. The plasma in a stellarator, as in several other systems using magnetic confinement, is subject to various instabilities causing a movement of the plasma to the walls and a consequent loss of energy. One of these instabilities, which is similar to the kink instability of a pinched discharge (§3.33), can arise when the current flowing around the stellarator, e.g., for heating the plasma, exceeds a certain value. Such a current produces an azimuthal magnetic field similar to that responsible for the pinch effect described above but usually of a much lower strength. This field tends to oppose the displacement effect on the lines of force produced by distortion of the torus in a stellarator system. When the current is equal to or greater than that value which would reduce the displacement in a single circuit to zero, kinklike perturbations in the plasma can grow until the plasma touches the walls of the container.

3.40. Another type of instability to which the plasma contained in a twisted torus confined by an axial magnetic field is subject is the interchange instability. It is also called the "flute type" instability since it is associated with the development of perturbations on the plasma surface which resemble the flutes of a column. When this occurs in such a way that the interchange of lines of force between the plasma and the surrounding vacuum field leads to a decrease in energy, the plasma is unstable. Theory indicates, however, that the interchange stability should be largely suppressed by the same helical magnetic field that, in some forms of the stellarator, is used to displace the field lines.

3.41. While the charged particles move around the stellarator, the gradient of the magnetic field in the curved ends causes them to drift normal to the field direction and to the field gradient. The tendency is thus for the particles of a given sign to drift upward in one loop and downward in the other loop of the stellarator. The tendency toward charge separation is counteracted by currents flowing along the lines of force, as indicated above. These currents produce a secondary magnetic field in a plane at right angles to the axis of the straight sections of the stellarator which is proportional to the density of the plasma. Some of the lines of force thus intersect the walls of the tube and lead to a decrease in confinement of the plasma.

3.42. As a consequence of this effect, the kinetic pressure of the particles which can be confined by the applied field is considerably less than the maximum value of $B_0^2/8\pi$ given by equation (3.19). In other words, a much stronger magnetic field is required to confine a plasma having a given particle density than would otherwise have been the case. Since the power of a thermonuclear reactor is proportional to the square of the particle density, this is a serious drawback. A possible method for overcoming the effect of the secondary currents, which involves special shaping of the stellarator tube, will be described in Chapter 8.

3.43. An important aspect of the stellarator is that it is possible and would be desirable to operate it continuously, instead of intermittently as appears to be necessary for pinched-plasma systems. The preference for continuous operation arises from the fact that in the stellarator there is no appreciable contraction of the plasma and so it is not easy to convert the thermonuclear energy directly into electric power by expansion of the plasma in a magnetic

field. Consequently, there is nothing to be gained from a pulsed operation, whereas there is much to be lost because the thermonuclear reactions do not occur in the intervals between pulses. Thus, for the stellarator, continuous operation, probably with an equimolar mixture of deuterium and tritium as fuel (§2.88), would be advantageous.

MAGNETIC MIRROR SYSTEMS

3.44. The third proposed method of magnetic confinement of a plasma to be described here avoids the problems of drift in an endless tube by employing a straight tube. A longitudinal magnetic field, for confinement, is applied externally by means of a solenoid, but instead of being uniform, the field strength is increased at the ends of the tube (Fig. 3.4). The region of

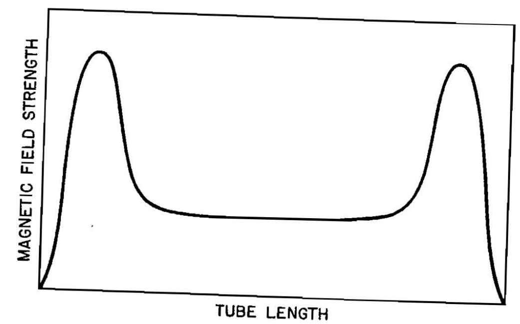

FIG. 3.4. Variation of magnetic field in magnetic mirror system.

enhanced magnetic field is referred to as a *magnetic mirror* (§9.1 *et seq.*); under suitable conditions charged particles, moving from the region of lower to that of higher field strength, will be reflected back into the former region. The magnetic field shown in Fig. 3.4 thus acts as a sort of potential well which inhibits the escape of many of the charged particles (and loss of energy) at the ends of a cylindrical tube.

3.45. The condition for reflection to occur is that in the region between the mirrors the ratio of $W_{\parallel}/W_{\perp}$, where $W_{\parallel}$ and $W_{\perp}$ are the components of the particle energy parallel and perpendicular, respectively, to the magnetic lines in the central field, should be equal to or less than $R - 1$, where R is the mirror ratio, i.e., the ratio of strengths of the stronger and weaker fields. The physical interpretation of this result may be understood with the aid of Fig. 3.5, which shows some of the magnetic lines of force in a straight tube with mirrors at the ends. Where the field is stronger, i.e., at the mirrors, the lines crowd closer together, so that they change direction in the region in which the field strength is increasing. The force exerted by a magnetic field on a charged particle is always in the direction perpendicular to the field lines, as shown by the arrows in the figure. It is seen, therefore, that, as a result of the curvature of the lines of force in the mirror region, the direction of the magnetic force is such that there is a backward component tending to push the particle back into the region between the mirrors. In other words, the effect of the mirror field is to decrease the longitudinal component of the particle velocity, i.e., the velocity component parallel to the direction of the field in the central region. If the original velocity vector is such that this component is reduced to zero somewhere in the mirror region, the particle will be reflected and will not escape.

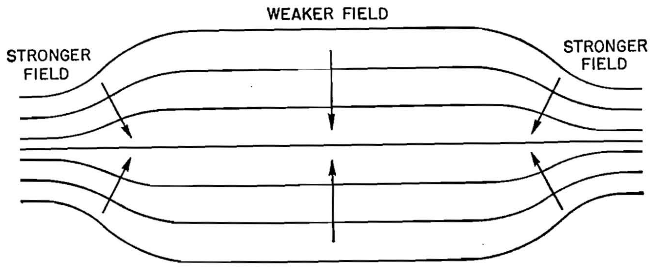

FIG. 3.5. Field lines and direction of force in magnetic mirror system.

3.46. It is apparent, therefore, that only particles with kinetic energy due primarily to their motion perpendicular to the central magnetic field, i.e., to the tube axis, will be confined in the mirror system described above. If the energy is due mainly to motion parallel to the central field lines, they will not be reflected at the ends and will escape through the mirrors. Even if all the charged particles initially in the plasma satisfied the condition for reflection, scattering collisions among the particles will inevitably occur. The partial randomization of the motion will thus mean that some of the particles will then not be reflected and will consequently be lost when they enter the mirror region.

3.47. Several schemes have been proposed for minimizing the losses due to escape from the magnetic field. One, for example, is to increase the value of the mirror ratio. Another possibility is to operate at temperatures greatly in excess of the ideal ignition temperature (§2.68). By increasing the temperature, the reaction cross sections are increased, but the scattering cross sections are decreased. The probability of energy gain due to thermonuclear reaction is thus increased relative to that of loss at the ends of the tube.

3.48. In order to supply energy to the plasma, it may be necessary to compress it adiabatically (§5.18). Axial (or longitudinal) compression can be achieved by moving the magnetic mirrors closer together, either mechanically or electrically. To obtain radial compression, the strength of the magnetic field between the mirrors is increased by increasing the current through the solenoid. Preferably both the mirror and central fields should be increased

proportionately, so that the mirror ratio remains constant. When the thermonuclear reaction occurs sufficiently rapidly, so that more energy is being produced than is lost in one way or another, the plasma can expand against the confining magnetic field and electricity could be generated directly in the external coils.

3.49. Although a plasma confined in a mirror system is subject to certain instabilities, e.g., the flute-type instability, they appear, at least at low plasma densities, to be much less serious than those occurring in a pinched discharge or in a stellarator. However, there are other problems which are not encountered in the latter devices. One of these, to be considered more fully in later chapters, is concerned with the production and injection of the plasma between the mirrors. Another difficulty may arise from the need for operating at temperatures considerably in excess of the minimum theoretical value. The particle density of the plasma which can be contained by the available magnetic fields will then be relatively low, resulting in a small power output. If the temperatures required prove to be too high to be practical with deuterium alone, it may be necessary to use a deuterium-tritium mixture.

THE ASTRON SYSTEM

3.50. Another proposed method for confining a plasma by means of a magnetic field is based on the *Astron concept* which is described more fully in Chapter 10. An axial magnetic field is produced in a long evacuated cylindrical chamber by means of a solenoidal coil, in the usual manner; then a beam of high-energy electrons, probably 30 to 50 Mev energy in an actual thermonuclear reactor, is injected at one end. As a result of the action of the magnetic field, the electrons are expected to form a circulating layer of current about the central axis. When the current in this layer, called the E-layer, reaches a certain value, the configuration of the magnetic field will change so as to form a pattern of closed magnetic lines of force, as shown in Fig. 3.6.

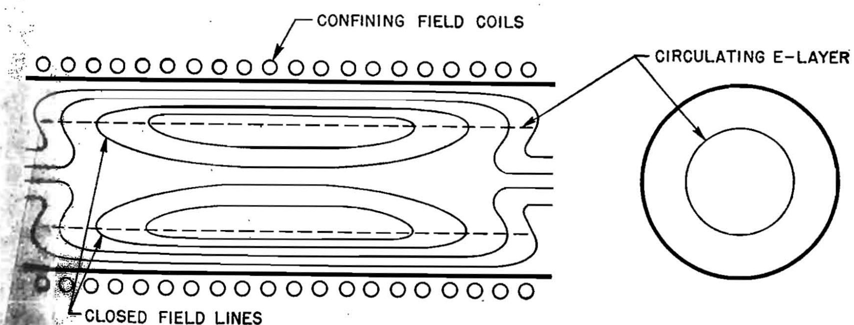

FIG. 3.6. Magnetic field configuration in Astron system.

This field can provide confinement of a plasma since the charged particles can escape only by the slow process of diffusion across the field lines.

3.51. If cold deuterium gas or, preferably, a mixture of deuterium and tritium is now injected, it will be immediately ionized by the high-energy electrons of the E-layer. The resultant plasma will be trapped by the closed system of magnetic field lines formed by the combination of the current in the external field coils and the E-layer current. The charged particles in the plasma will gain energy by collisions with the energetic electrons, so that temperatures may be attained at which thermonuclear D-T reactions take place at a useful rate.

3.52. A theoretical analysis indicates that the plasma in an Astron system should be stable against the more catastrophic perturbations. However, the concept involves such novel ideas, concerning which there has been little practical experience, that only the experimental investigations now proceeding can provide answers to the problems involved.

FUSION REACTIONS WITH ACCELERATED PARTICLES

BOMBARDMENT OF A SOLID TARGET

3.53. Apart from the proposal to inject deuterons of high energy, e.g., 300 kev or more, produced by a conventional accelerator, into a magnetic mirror system where they assume an approximately Maxwellian energy distribution before combining, a number of other suggestions have been made for initiating fusion reactions by the use of accelerated particles. Some of these proposals will be examined below [1].

3.54. An obvious method for bringing about a D-D or D-T reaction is to bombard a solid target containing deuterium or tritium with a beam of high-energy deuterons. In this case, most of the energy of the accelerated particles is expended uselessly in ionization of the target atoms. It will be shown in Chapter 4 that the Coulomb scattering cross section for the close-range interaction of a high-energy (or "hot") deuteron, having an energy of 100 kev, with a low-energy (or "cold") electron in the target exceeds 10^7 barns (§4.52) compared with 0.03 barn for the D-D reactions and 5 barns for the D-T reaction. It is apparent, therefore, that the Coulomb scattering of electrons, leading to ionization of the target atoms, will occur far more frequently than fusion reactions leading to the release of energy [1, 6].

3.55. For the bombardment of deuterium by 100-kev deuterons, for example, a comparison of the cross sections given above shows that there would be roughly 3.3×10^8 scattering collisions for every one leading to reaction. The fraction of energy lost by a deuteron in each scattering collision with an electron is roughly equal to the ratio of the mass of the electron to that of the deuteron, i.e., about 1/3660. In the 3.3×10^8 scattering collisions between 100-kev deuterons and cold electrons, the energy loss is thus $3.3 \times 10^8 \times 100 \times 1/3660 \approx 9 \times 10^6$ kev or 9×10^3 Mev. The average energy produced in the one reactive D-D collision is about 12.5 Mev (§2.23), so that far more energy is used in accelerating the particles than is recovered by the fusion reactions. For a tritium target bombarded by deuterons, the situation is somewhat more favorable, since in a D-T mixture reaction there are roughly 2×10^6 collisions for each one leading to reaction. In this case, however, it still is necessary to expend about 50 Mev to recover 14 Mev of fusion energy.

3.56. In the foregoing calculation, no allowance has been made for the fact that the scattered incident deuteron may still have enough energy to interact with a deuterium (or tritium) nucleus in the target. However, it is known that, as the energy of the deuteron decreases, the scattering cross section increases whereas the reaction cross section decreases. Another factor which has not been taken into account is multiple small-angle scatterings of the deuteron. Although this may not necessarily lead to ionization by ejection of electrons, it will result in considerable loss of energy by the deuteron.

BOMBARDMENT OF A PLASMA TARGET

3.57. A possible modification of the procedure described above, and one which would represent a decided improvement, is to use a plasma from an ordinary gas discharge as the target. In this way the problem of energy losses due to ionization of the target atoms would be largely eliminated. However, an ordinary gas discharge, in which the electron temperatures are of the order of 10 ev, offers little hope of achieving a net gain of energy as a result of fusion reactions. Since the 10-ev electrons are cold in comparison with the hot incoming ions, with energies of 100 kev or more, it appears that in these circumstances, as will be shown below, the transfer of energy from the hot deuteron to the cold electron is much more probable than reaction of the deuteron with a deuterium or tritium nucleus in the gas. Furthermore, as a result of this energy transfer the deuteron energy is reduced to a value at which the D-D or D-T reaction cross section is very small.

3.58. At a particle density of 10^{15} electrons/cm^3 and an electron temperature of 10 ev, the time required for 100-kev deuterons to transfer most of their energy to the electrons is about 10^{-6} sec (§4.82). This is very short in comparison with the D-D mean reaction time of approximately 40 sec or even with the 1 sec for the D-T reaction at the given energy and a density of 10^{15} deuterons/cm^3 (§2.38). Thus, most of the energy of the bombarding deuterons would be utilized in raising the temperature of the electrons.

3.59. If the initial temperature of the electrons in the plasma were considerably higher than the 10 ev assumed above, the situation might be very different [7]. Calculations indicate that bombardment of a deuterium plasma in which the electron temperature was 10 kev, and possibly as low as 1 kev, by accelerated tritons having energy in the range of 100 to 300 kev could lead to a considerable net gain of energy. The temperature of the deuterons in the plasma target apparently does not need to exceed a few hundred electron volts. The deuterium plasma would of course have to be confined by a magnetic field, e.g., in a stellarator-type system. Initial preferential heating of the electrons could be readily achieved in a conventional manner by passage of an electrical discharge through the ionized gas (§5.9).

HEATING BY ACCELERATED PARTICLES

3.60. When an accelerated particle strikes a cold target and loses most of its energy in causing ionization, much of the lost energy will shortly appear in the form of heat. For example, the ionized atoms (or nuclei) in the target will recapture the electrons removed in the ionization process and emit energy as radiation. In a solid target, especially, the radiation will be readily absorbed and converted into heat. It may be wondered, therefore, if thermonuclear reactions might not be brought about by utilizing the heat generated locally by the accelerated particles. However, the following calculation, based on the most optimistic assumptions, shows that the temperature attained would be far below the minimum value of 10 kev, at least, required for the production of more energy by thermonuclear fusion than is lost by radiation as bremsstrahlung.

3.61. Suppose a beam of 1-Mev deuterons impinges on a solid target. It will be assumed that the beam current density is of the order of 100 amp/cm^2, which is probably higher than is actually attainable because of space-charge effects. Since 1 amp is 3×10^9 statamp and the charge carried by a deuteron is 4.8×10^{-10} statcoulomb, the particle flux in the beam is $(100 \times 3 \times 10^9)/(4.8 \times 10^{-10}) \approx 6 \times 10^{20}$ deuterons/(cm^2)(sec). The deuteron energy is 1 Mev, i.e., 10^6 ev, and so the energy flux of the beam is 6×10^{26} ev/(cm^2)(sec).

3.62. The range of a 1-Mev deuteron in matter is about 1.5 mg/cm^2. Hence, the mass of deuterium exposed to the accelerated deuterons in an area of 1 cm^2 is 1.5×10^{-3} g. Since 2 g of deuterium contain 6.0×10^{23} (the Avogadro number) atoms, the total number of particles, both nuclei and electrons, in the target volume is given by

$$\text{Number of particles in the target volume} = 2 \times \tfrac{1}{2} \times 6.0 \times 10^{23} \times 1.5 \times 10^{-3} = 9 \times 10^{20}.$$

Suppose the time an accelerated deuteron and a target particle spend in the vicinity of each other is 10^{-6} sec; this is probably longer than the actual time, but it will be used here, since it leads to results which are more favorable than can be expected. During this time the energy falling on 1 cm^2 is $6 \times 10^{26} \times 10^{-6} = 6 \times 10^{20}$ ev. If this is shared equally among the target particles, it follows that

$$\text{Energy per target particle} = \frac{6 \times 10^{20}}{9 \times 10^{20}} \approx 0.7 \text{ ev}.$$

This energy is of course much too low to satisfy the requirements for obtaining net power from nuclear fusion. No reasonable changes in the conditions postulated above for the purpose of the calculation could lead to an energy per particle in the required kilo-electron volt range [6].

3.63. Since, in the procedure just outlined, the accelerated deuterons are merely used to raise the temperature of the target nuclei and need not take part in any reaction, other accelerated ions could be used for this purpose. With nuclei of higher atomic number, more energy could be transferred to the target particles. Nevertheless, it is not apparent how conditions for useful nuclear fusion could be attained in this manner.

COLLIDING BEAMS OF ACCELERATED PARTICLES

3.64. Another method for utilizing accelerated deuterons which has been proposed from time to time is that two beams of deuterons (or one of deuterons and one of tritons) be directed at each other, thus obtaining reactions as a result of mutual collisions. This suggestion has the merit of apparently solving the problem of increasing the relative collision energy and thus increasing the effective cross section for the reaction. Furthermore, by confining all the reacting nuclei to beams, heat losses, e.g., to the walls of the vessel, are reduced, temporarily at least.

3.65. Like some of the other schemes for utilizing beams of accelerated particles proposed above, this one can also be shown to fail when considered quantitatively. There are two main reasons for this situation: in the first place, the cross section for large-angle scattering of deuterons by deuterons is several thousand times as great as that for the D-D reaction (§4.62). Consequently, most of the accelerated particles would be scattered out of the beam without reacting. The fusion energy released by the relatively few deuterons that did succeed in reacting would be only a small proportion of that utilized to accelerate them.

3.66. The second factor making the method of colliding beams unattractive for the production of useful power is the very low maximum particle density which can be attained and the consequent small power density [1]. Suppose the energy of the deuterons in each beam is 50 kev, so that 100 kev is the relative energy in a collision. The ion current density may be taken as having a value of 100 amp/cm^2, corresponding to 6×10^{20} deuterons/(cm^2)(sec). This is probably much larger than is actually attainable. The mean velocity of a 50-kev deuteron is about 2×10^8 cm/sec, and so the particle density in the accelerated beam is 3×10^{12} deuterons/cm^3. Utilizing the cross section for the D-D reactions at 100 kev, from Fig. 2.3, it is found that the power density in the colliding beams would be about 10^{-4} watt/cm^3. For the D-T reaction, the corresponding power density would be approximately 2×10^{-2} watt/cm^3. These are too small to be of any practical value. Increasing the acceleration energy to 500 kev, so that 1 Mev is available per collision, would increase the power density for the D-D reaction to 7×10^{-4} watt/cm^3, whereas that for the D-T reaction would be less than at 100 kev because of the decrease in the cross section (see Fig. 2.3).

REFERENCES FOR CHAPTER 3

1. R. F. Post, *Rev. Mod. Phys.*, **28**, 338 (1956).
2. J. L. Craston, R. Hancox, A. E. Robson, S. Kaufman, H. T. Miles, A. A. Ware, and J. A. Wesson, *Proc. Second U.N. Conf. on Peaceful Uses of Atomic Energy*, **32**, 414 (1958).
3. A. E. Robson and R. Hancox, *Proc. Inst. Elec. Eng. (London)*, **106A**, Suppl. No. 2, 47 (1959).
4. G. P. Harnwell, *Principles of Electricity and Magnetism*, McGraw-Hill Book Co., Inc., 1949, p. 63, footnote.
5. R. F. Post, *Ann. Rev. Nuclear Sci.*, **9**, 367 (1959).
6. J. D. Lawson, UKAEA Report AERE GP/M-185 (1955).
7. W. I. Linlor, Hughes Aircraft Co. Research Report 128 (1959).

Chapter 4

ELEMENTARY PLASMA THEORY

INTRODUCTION

PLASMA PROPERTIES

4.1. It was seen in Chapter 3 that the hopes for confining a plasma consisting almost entirely of hydrogen isotope nuclei and electrons at high temperatures rest mainly in the use of magnetic fields. To understand and overcome the many problems arising in the implementation of the ideas reviewed in the preceding chapter, a detailed knowledge, both theoretical and experimental, is required of the behavior of plasmas in electromagnetic fields [1-5].

4.2. The gross behavior of a plasma is governed by interactions among the charged particles themselves and between the particles and external electromagnetic fields. Strictly speaking these interactions are not independent, since the plasma may profoundly affect the character of an external applied field through collective effects. Nevertheless, by treating the processes separately, it is often possible to derive a reasonable approximation to the actual behavior of a plasma.

4.3. In the present chapter it is proposed only to examine some of the simpler theoretical aspects of the properties and behavior of plasmas such as are necessary for a general understanding of the topics to be considered in later chapters. The discussion here is divided into three main parts: (1) the motion of individual charged particles in electromagnetic fields; (2) the interactions among the particles in a plasma; and (3) certain cooperative (or collective) properties of the plasma as a whole.

THEORETICAL PLASMA MODELS

4.4. The theoretical treatment of a plasma may be undertaken from several points of view, that is, by using different models of the plasma. The model used in any particular circumstance will depend on the property being examined and on the physical conditions assumed for the problem being considered. Two of the most important problems in the study of controlled thermonuclear reactions are confinement of the plasma by magnetic fields and the stability of various plasma-field configurations. It is convenient in these cases to use either of two simplified pictures of the ionized gas. The first is called the *hydromagnetic model*, in which the plasma is treated as a compressible conducting fluid that is subjected to the action of electromagnetic forces. In the second, known as the *single-particle model*, the system behavior is obtained by assuming that each charged particle is acted upon individually by the externally applied field and that collision effects, which would lead to interaction among the particles, are minor.

4.5. The hydromagnetic model is of greatest utility in studying the stability of the plasma in magnetic fields of various configurations and of the closely related problem of hydromagnetic waves in a plasma. The calculations require only a knowledge of the magnetic field configuration and of the density and possibly the conductivity of the plasma, which is treated as an ideal hydrodynamic fluid (see Chapter 13).

4.6. For the investigation of confinement by a magnetic field, both the hydromagnetic and single-particle models may be used to provide results of interest. A simple example of the hydromagnetic method is the treatment in Chapter 3 which led to the derivation of equation (3.15) relating the strength of a uniform magnetic field to the maximum pressure of a confined plasma in a steady-state system. In the present chapter, the single-particle approach will be used to provide information concerning the motion of charged particles in electromagnetic fields since this has a bearing on the problem of confinement.

4.7. The rigorous starting point, from which the two models described above appear as approximations, is the Boltzmann equation for the distribution of particles in phase space as a function of position and velocity (see §13.40). As applied to a plasma, the force term in the ordinary form of the Boltzmann equation is set equal to the electromagnetic force on a charged particle. By taking successive moments of the Boltzmann equation, i.e., upon multiplying by various functions of the velocity and integrating over velocity space, equations of particle conservation, momentum transfer, and energy transfer are obtained. The momentum transfer equation may then be subjected to various approximations in order to obtain the equations of motion for the hydromagnetic and single-particle models of a plasma.

4.8. The treatment of intensive properties of a plasma, e.g., diffusion coefficient, thermal and electrical conductivities, radiation effects, and energy transfer between two particles, ordinarily requires detailed analysis of the collisions in an electron-ion gas from the standpoint of kinetic theory modified to take into account Coulomb forces. Some of these properties will be treated in this chapter (§4.48 *et seq.*) and others will be discussed in Chapter 12. If external electromagnetic fields are imposed on the system, they are usually treated as perturbations on the properties which are determined essentially by collisions.

MOTION OF CHARGED PARTICLES IN ELECTRIC AND MAGNETIC FIELDS

INTRODUCTION

4.9. In principle, the behavior of a plasma in an electromagnetic field can be determined by finding a solution for the motion of each plasma particle in the field produced by all its neighbors combined with that applied externally. This represents, essentially, a self-consistent application of both single-particle and hydromagnetic models of the plasma. Such an approach is generally too complicated for satisfactory analysis, but in the limit of relatively strong magnetic fields and low plasma densities, such as would probably exist in a thermonuclear reactor, useful information can be obtained simply by considering the motion of a single particle in the applied electric and magnetic fields [3, 4, 6-9].

4.10. It has been mentioned earlier that in a magnetic field a charged particle gyrates about the field lines; the center of gyration at any instant is called the *guiding center* of the particle. As the gyrating particle moves along a line of force in a uniform field, it will follow a helical path, but its guiding center will remain on the field line. However, if the magnetic field is not homogeneous, if there is an applied electric field, or if gravitational forces are significant, various drift motions, which can be expressed as motions of the guiding center, will be superimposed on the normal helical path of the charged particle.

4.11. In determining the motion of the guiding center, the first step is to derive a general expression for the force acting on a particle as a result of its motion in electric and magnetic fields. The equation of motion (or force) for a single charged particle of mass m and charge $\pm e$, moving with a velocity v and exposed to the action of an electric field $\mathbf{E}$ and a magnetic field $\mathbf{B}$, can be derived in a manner exactly similar to that employed in §3.15. If the forces acting on the charged particle are equated to its rate of change of momentum, the result, expressed in Gaussian-cgs units, is

$$m\frac{d\mathbf{v}}{dt} = e\left[\mathbf{E} + \frac{1}{c}(\mathbf{v} \times \mathbf{B})\right]. \tag{4.1}$$

This equation implies that, if the electric field has a component parallel to the motion of the charged particle, the latter moves with a constant acceleration $e\mathbf{E}/m$, so that its kinetic energy increases continuously. The magnetic field, on the other hand, exerts its force in a direction perpendicular to both the direction of motion and that of the field lines; there is then no force along the direction of motion. Consequently, the magnetic field produces a curvature in the path of the particle, but there is no change in its scalar velocity, and hence its kinetic energy is unaffected by the field. Equation (4.1) has simple solutions in a number of special cases which will now be considered.

MOTION IN A UNIFORM MAGNETIC FIELD

4.12. For the case in which there is no electric field and **B** is constant in space and time, equation (4.1) reduces to

$$m\frac{d\mathbf{v}}{dt} = \frac{e}{c}(\mathbf{v} \times \mathbf{B}). \qquad (4.2)$$

Consequently, the acceleration $d\mathbf{v}/dt$ is perpendicular to the velocity; this means that the velocity does not change in magnitude but only in direction. The particle thus moves in a circular path with a constant velocity v while being acted upon by a force evB/c. If this force, due to the magnetic field, is equated numerically to the centrifugal force mv^2/r_g, where r_g is the radius of the circle, i.e.,

$$\frac{evB}{c} = \frac{mv^2}{r_g},$$

it is found that the angular frequency ω_g, which is equal to v/r_g, is given by

$$\omega_g = \frac{eB}{mc}. \qquad (4.3)$$

This is called the *gyromagnetic frequency* of the particle. The radius of the circle of gyration, known as the *gyromagnetic radius*, is

$$r_g = \frac{mvc}{eB}. \qquad (4.4)$$

If the particle under consideration is an electron or a hydrogen isotope ion, e is equal to the electronic charge, i.e., 4.80×10^{-10} esu.

4.13. The gyration frequency derived above is sometimes referred to as the *gyrofrequency*, and also as the *cyclotron frequency* because it is identical with the expression for the angular frequency of gyration of particles in a cyclotron. The adjective "gyromagnetic" is used in this book, however, as it is more completely descriptive of the situation than either of the others. Unfortunately, the terms Larmor frequency and Larmor radius have become widely, but incorrectly, used in plasma physics for the gyromagnetic frequency and radius, respectively. The Larmor angular frequency for the precession of an orbital electron in a magnetic field is $eB/2mc$, which is half the value of the frequency given by equation (4.3).

4.14. If the direction of motion of the particle is not initially perpendicular to the magnetic field, v in equation (4.4) must be replaced by $v_\perp$, the component perpendicular to **B**, so that in this case

$$r_g = \frac{mv_\perp c}{eB}. \qquad (4.5)$$

The component of v parallel to **B**, i.e., $v_\parallel$, will not be affected by the magnetic field. By superposing the motions perpendicular and parallel to field, it is seen that the particle will move with a constant velocity component along a magnetic line of force while gyrating around it at the angular frequency eB/mc. In other words, the orbit described by the charged particle is a helix of constant pitch, its axis being one of the field lines. Consequently, as stated earlier, in a uniform magnetic field the guiding center of the particle moves along a line of force.

4.15. According to equation (4.1), the direction of the force exerted by a magnetic field on a charged particle depends on the sign of the charge. Consequently, the positive ions (nuclei) and negative electrons in a plasma gyrate in opposite directions about the magnetic field lines. The gyromagnetic frequency of the ions in a given magnetic field is less than that of the electrons, because, as equation (4.3) shows, this frequency is inversely proportional to the mass, i.e., ω_i/ω_e is equal to m_e/m_i, where the subscripts e and i represent electrons and ions, respectively. For a deuteron, equation (4.3) gives the gyromagnetic frequency as

$$\omega_g = 4.8 \times 10^3 B \text{ radians/sec},$$

with B in gauss; the value for an electron is correspondingly greater by the factor of m_i/m_e, which is close to 3660 in this case.

4.16. The ratio of the gyromagnetic radii of ions and electrons depends on the velocities (or kinetic energies) of the particles perpendicular to the field lines. If the perpendicular component $W_\perp$ of the kinetic energy is the same for both ions and electrons, as would be the case if they had the same kinetic temperature, then $\frac{1}{2}mv^2_\perp$ is the same for both kinds of particles. It follows then from equation (4.5) that the ions execute orbits which are greater than the electron orbits by a factor of $(m_i/m_e)^{1/2}$. The product of magnetic field strength and the radius of gyration, a useful quantity experimentally, is given by equation (4.4) as

$$Br_g = \frac{mv_\perp c}{e}$$
$$= \frac{(2mW_\perp)^{1/2}c}{e}, \qquad (4.6)$$

which is equivalent to equation (3.21) for a Maxwellian distribution, since $W_\perp$ is then equal to kT. For a deuteron, equation (4.6) becomes

$$Br_g = 6.4 \times 10^3 W_\perp^{1/2} \text{ gauss-cm},$$

with $W_\perp$ in kilo-electron volts. The value of Br_g for an electron having the same value of $W_\perp$ is obtained upon dividing by $(m_i/m_e)^{1/2}$, which is approximately 60.

4.17. The time required for a particle to make a single turn in its gyration about the magnetic field lines is called the *gyromagnetic period* or *gyration* *time*, τ_g. It is equal to the length of a turn, i.e., $2\pi r_g$, divided by the velocity $v_\perp$ as given by equation (4.5); thus,

$$\tau_g = \frac{2\pi r_g}{v_\perp} = \frac{2\pi mc}{eB}. \qquad (4.7)$$

If Gaussian-cgs units are used, equation (4.7) gives τ_g in seconds.

MOTION IN ELECTRIC AND MAGNETIC FIELDS

4.18. When an electric field having a component perpendicular to the magnetic field is present, the particle path consists of a helical motion with a superposed drift at constant velocity in a direction perpendicular both to the magnetic field and to the component of the electric field. The actual motion may be regarded as arising from a transverse drift of the guiding center of the particle.

4.19. The mechanism which results in this drift of a charged particle across a magnetic field can be readily understood from a qualitative point of view. Consider a positively charged particle rotating about a line of force in a magnetic field, directed upward and perpendicular to the plane of the page, as

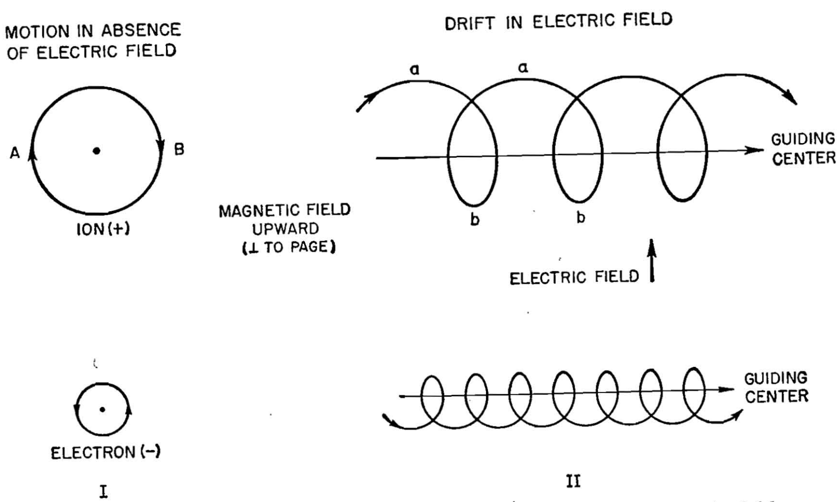

FIG. 4.1. Drift of charged particles in crossed electric and magnetic fields.

indicated in Fig. 4.1, I. If an electric field is applied, in the direction shown, i.e., from the bottom toward the top of the page, as seen in Fig. 4.1, II, so that it is perpendicular to the magnetic field, the positive ion will now be ac-

celerated as it moves in the region A and decelerated at B. According to equation (4.5), the radius of curvature of the path is proportional to the velocity. Consequently the radius is increased at a and decreased at b, in Fig. 4.1, II. The net result is that the ion follows a cycloidal path with the guiding center drifting to the right. For a negatively charged particle, e.g., an electron, the rotation in the same magnetic field is in the opposite direction to that of an ion, but it can be readily seen that the drift will also be to the right, as shown in the figure. Thus, the direction of drift is independent of the charge of the particle. Since the radius r_g of the helix is proportional to $mv_\perp$, by equation (4.5), it will be smaller for an electron than for an ion having the same rotational energy, i.e., perpendicular to the magnetic field direction.

4.20. In order to calculate the drift velocity of a charged particle, i.e., of its guiding center, in magnetic and electric fields at right angles, it will be supposed that the actual velocity $\mathbf{v}$ of the particle may be expressed by

$$\mathbf{v} = \mathbf{v}_0 + c\,\frac{\mathbf{E} \times \mathbf{B}}{B^2}, \qquad (4.8)$$

where the significance of the two terms on the right will be seen below. Suppose that both $\mathbf{E}$ and $\mathbf{B}$ are constant in space and time; substitution of equation (4.8) into (4.1) then leads to

$$m\frac{d\mathbf{v}_0}{dt} = e\left[\mathbf{E} + \frac{1}{c}(\mathbf{v}_0 \times \mathbf{B}) + \frac{1}{B^2}(\mathbf{E} \times \mathbf{B}) \times \mathbf{B}\right]. \qquad (4.9)$$

If, further, it is postulated that $\mathbf{E}$ is perpendicular to $\mathbf{B}$, so that $\mathbf{E}\cdot\mathbf{B}$ is zero, it is found that

$$(\mathbf{E} \times \mathbf{B}) \times \mathbf{B} = -B^2\mathbf{E},$$

and insertion of this result into the last term of equation (4.9) gives

$$m\frac{d\mathbf{v}_0}{dt} = \frac{e}{c}(\mathbf{v}_0 \times \mathbf{B}). \qquad (4.10)$$

This expression, which is similar to equation (4.2), shows that $\mathbf{v}_0$, as defined by equation (4.8), is independent of the electric field and consists of a gyration about the lines of force at the frequency ω_g.

4.21. If the movement of the particle may be regarded as consisting of a helical motion combined with a drift, it is evident that $\mathbf{v}_0$ represents the former. Hence, the drift velocity v_d is given by the last term on the right of equation (4.8); thus,

$$v_d = c\,\frac{\mathbf{E} \times \mathbf{B}}{B^2}$$

or, since it has been postulated that $\mathbf{E}$ and $\mathbf{B}$ are perpendicular,

$$v_d = c\,\frac{E}{B}. \qquad (4.11)$$

By expressing E in volts/cm, instead of in statvolts/cm, the result is

$$v_d \text{ (cm/sec)} = \frac{10^8 E \text{ (volt/cm)}}{B \text{ (gauss)}}. \tag{4.12}$$

4.22. It is seen that the drift velocity is independent of the mass and initial velocity of the charged particle. As was stated above, the direction of drift is also independent of the sign of the charge. It should be mentioned that equation (4.11) is applicable provided the calculated value of the drift velocity v_d is less than the velocity of light. For a magnetic field of 10^5 gauss, for example, the results would be valid provided the electric field strength is less than the very large value of 3×10^7 volts/cm.

MOTION IN A TIME-VARYING MAGNETIC FIELD

4.23. An important application of equation (4.11) for the drift velocity is in connection with a magnetic field which varies with time. Suppose that a uniform magnetic field, provided by a long solenoid, is increasing with time. In these circumstances, an electric field appears, consisting of circular lines of electric force centered on the axis of the solenoid. The magnitude E of this field, in the laboratory frame of reference, can be determind by means of the electromagnetic induction equation

$$\oint E dl = -\frac{1}{c}\int \left(\frac{dB}{dt}\right) dA,$$

where dl is an element of length around the boundary of the contour of an element of area dA. For the case of a circular contour of radius r centered on the solenoid axis,

$$2\pi r E = -\frac{\pi r^2}{c}\cdot\frac{dB}{dt},$$

so that

$$E = -\frac{r}{2c}\cdot\frac{dB}{dt}. \tag{4.13}$$

4.24. For a plasma of low density (or high temperature), especially in a strong magnetic field, the gyromagnetic frequency is much greater than the frequency of collisions which tend to shift the guiding centers of the particles across the field lines.* The charged particles may then be regarded as being tightly bound to the lines of force. In that event the particle drift velocity in a time-varying magnetic field is represented by dr/dt; hence, by equation (4.11), this may be set equal to cE/B, and it then follows from equation (4.13) that

$$\frac{2}{r}\cdot\frac{dr}{dt} = -\frac{1}{B}\cdot\frac{dB}{dt}.$$

* When collisions are relatively frequent, the particles are able to cross the magnetic field lines and then diffusion can occur; this is discussed in Chapter 12.

Upon integration this leads to the result

$$r^2 B = \text{constant}. \tag{4.14}$$

4.25. The conclusion to be drawn from equation (4.14) is that, in the radial motion of any particle with reference to the solenoid axis, in a time-varying magnetic field, the total flux between the moving point and the axis remains constant. In other words, the guiding centers of particles moving at the local drift velocity remain on the surface of some collapsing (or expanding) flux tube of the changing magnetic field. Another way of stating this conclusion is that the charged particles tend to stick to the lines of force.

MOTION IN AN INHOMOGENEOUS MAGNETIC FIELD

4.26. In the presence of a magnetic field with strength varying in space, so that magnetic gradients exist, the motion of a charged particle is somewhat complex. When the gradients are small, the motion can be represented by simple relationships. The case of a magnetic field with a gradient perpendicular to the local direction of the field lines is of particular interest.* The effects on a positive ion and an electron are shown qualitatively in Fig. 4.2; the magnetic field is directed upward as in Fig. 4.1, and the gradient is

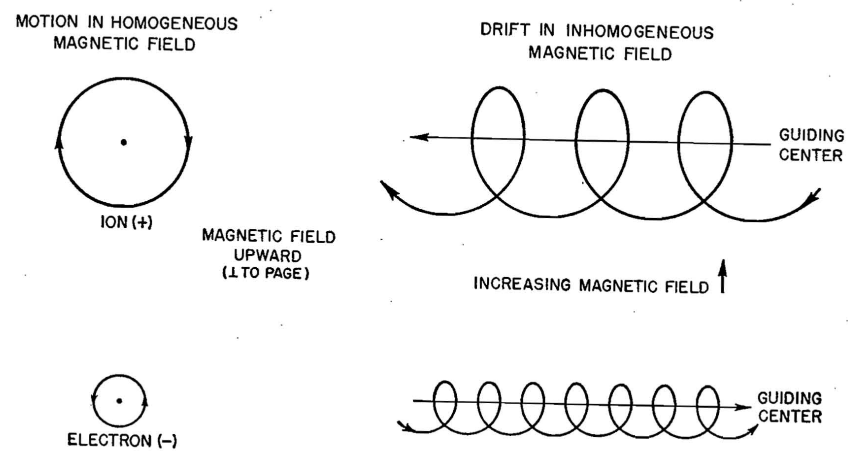

FIG. 4.2. Drift of charged particles in inhomogeneous magnetic field.

in a perpendicular direction, i.e., in the plane of the page. Where the field is stronger, the radius of curvature of the path is smaller than average, in accordance with equation (4.5); on the other hand, where the field is weaker, the radius is greater. The result is that the charged particles follow a cycloidal

* When the magnetic field gradient is parallel to the field lines, the situation corresponds to a "magnetic mirror," which will be discussed in Chapter 9.

path, but the guiding centers of the positive and negative particles now drift in opposite directions perpendicular both to the magnetic field and the gradient.

4.27. In a plasma located in an inhomogeneous magnetic field, therefore, the tendency for the ions and the electrons to drift in opposite directions would result in a charge separation. This, in turn, would produce a local electrostatic (space-charge) field of large magnitude. Such a field acting in conjunction with the magnetic field at right angles causes the ions and electrons, i.e., the plasma as a whole, to drift perpendicular to both fields, in the direction of decreasing magnetic field.

4.28. The derivation of the drift velocity of an individual particle in an inhomogeneous magnetic field is complicated [10]; however, the following approximate method, based on a simple physical model, leads to a substantially correct result [11]. Suppose the magnetic field acts in a direction perpendicular to and out of the plane of the page, and that the charged particle moves in this plane. Instead of a gradual variation in the strength of the inhomogeneous magnetic field, the simple case will be considered in which the field strength changes sharply from B_1, at the left of the vertical line in Fig. 4.3, to B_2, at the right of the line. If the particle starts at a and makes half

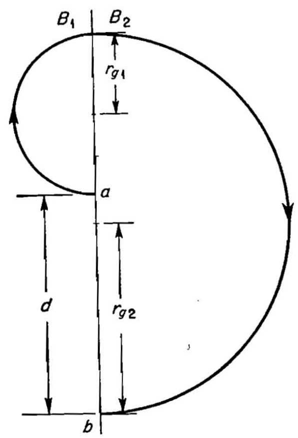

FIG. 4.3. Calculation of drift velocity in inhomogeneous magnetic field.

a gyration in the field B_1 and the other half in B_2, it will finish up at b. Hence, the net distance drifted will be d, which is equal to $2(r_{g2} - r_{g1})$, where r_{g1} and r_{g2} are the respective gyromagnetic radii in the two fields. The time required to traverse this distance is $\frac{1}{2}(\tau_1 + \tau_2)$, where τ_1 and τ_2 are the respective gyromagnetic periods. The drift velocity v_d can then be represented, approximately, by

$$v_d \approx \frac{2(r_{g2} - r_{g1})}{\frac{1}{2}(\tau_1 + \tau_2)}.$$

Upon substituting the appropriate expressions for r_g and τ, using equations (4.5) and (4.7), respectively, it is found that

$$v_d \approx \frac{2v_\perp}{\pi} \cdot \frac{B_1 - B_2}{B_1 + B_2}. \tag{4.15}$$

4.29. The average distance of a semicircular path from the diameter is equal to $\frac{1}{4}\pi$ times the radius. Hence, in the present case, the gradient of the magnitude of the field in the plane perpendicular to the field direction may be represented by

$$\nabla_\perp B \approx \frac{B_1 - B_2}{\frac{1}{4}\pi(r_{g2} + r_{g1})},$$

so that

$$B_1 - B_2 \approx \tfrac{1}{4}\pi(r_{g2} + r_{g1})\,\nabla_\perp B$$
$$= \frac{\pi}{4} \cdot \frac{mv_\perp c}{e} \frac{B_1 B_2}{B_1 + B_2} \nabla_\perp B.$$

Furthermore, if B is the average field strength,

$$B_1 + B_2 = 2B \quad \text{and} \quad B_1 B_2 \approx B^2.$$

Upon making the appropriate substitutions into equation (4.15), it is found that

$$v_d \approx \frac{\frac{1}{2}mv_\perp^2 c}{eB^2} \nabla_\perp B = \frac{W_\perp c}{eB^2} \nabla_\perp B, \tag{4.16}$$

where $W_\perp$ is the kinetic energy component perpendicular to the field lines. A more rigorous treatment shows that equation (4.16) is valid only if $v_d \ll v_\perp$, i.e., if $\frac{1}{2}r_g(\nabla_\perp B/B) \ll 1$; this condition implies that the fractional change in magnetic field strength across an orbit must be small.

EFFECT OF GRAVITATIONAL AND CENTRIFUGAL FORCES

4.30. When a charged particle in a homogeneous magnetic field is subjected to a gravitational force which has a component $mg_\perp$ perpendicular to $\mathbf{B}$, a drift will result. The situation is similar to that arising from an applied electric field $\mathbf{E}$, except that the gravitational force $mg_\perp$ replaces the force $\mathbf{E}e$. The drift velocity in the gravitational field is then found by substituting $mg_\perp/e$ for E in equation (4.11); hence,

$$v_d = \frac{mg_\perp c}{eB}. \tag{4.17}$$

If equation (4.17) is combined with equation (4.3) for the gyromagnetic frequency ω_g of the particle, the result is

$$v_d = \frac{g_\perp}{\omega_g}. \qquad (4.18)$$

The direction of drift is perpendicular to both the magnetic and gravitational fields. Contrary to the behavior in an electric field, ions and electrons now drift in opposite directions. This is because they gyrate in opposite directions and the effect of the gravitational force is independent of the sign of the charge. Since, for deuterons (§4.15), ω_g is equal to $4.8 \times 10^3 B$, with B in gauss, it is evident that g/ω_g is very small for magnetic fields in the kilogauss range, and so the gravitational drift velocity is negligible in situations of interest.

4.31. If a particle is gyrating about a line of force which is curved, a centrifugal acceleration will arise which results in a drift similar to that due to a gravitational force. The drift velocity can in fact be obtained upon replacing $g_\perp$ in equation (4.17) by the centrifugal acceleration. Suppose the particle is executing a helical path, with the guiding center moving along a curved line of force, with a velocity component of $v_\parallel$ along this line. Then, if r is the radius of curvature of the magnetic field, the centrifugal acceleration is $v_\parallel^2/r$ and

$$v_d = \frac{mv_\parallel^2 c}{eBr}$$

$$= \frac{2W_\parallel c}{eBr}, \qquad (4.19)$$

where $W_\parallel$ is the kinetic energy component of the particle in the direction of the field line.

4.32. In contrast to gravitational drifts, which are generally small, the drift velocity due to curvature of the magnetic lines of force may be substantial. However, as in gravitational and inhomogeneous magnetic fields, the ions and electrons tend to drift in opposite directions. This may consequently give rise to charge separation and the development of an appreciable electrostatic (space-charge) field. The result, as mentioned above in connection with inhomogeneous magnetic fields, is that the plasma will move as a whole in the direction of the weaker magnetic field.

PARTICLE DRIFT IN A TOROIDAL MAGNETIC FIELD

4.33. A consequence of fundamental significance in connection with the achievement of controlled thermonuclear reactions can be derived from the results presented above. As stated in §3.14, it has been suggested that energy losses which would normally occur at the ends of a tube containing a plasma confined by a simple, axial magnetic field could be avoided by bringing the ends together to form a closed torus. Unfortunately, this is not a satisfactory solution to the problem because the magnetic field in the torus is both curved and nonuniform and as a result the plasma would drift to the walls.

4.34. Consider a torus containing a plasma in which a magnetic field is produced by passage of current through a solenoid wound around the torus (Fig. 4.4). In the absence of an electric field (or in the presence of an electric field that is constant in time) the Maxwell equation (3.8) for the curl of the magnetic field strength may be written as

$$\nabla \times \mathbf{B} = \frac{4\pi}{c}\mathbf{j}, \qquad (4.20)$$

where $\mathbf{j}$ is the current density in the solenoid. Integrating both sides of this equation over the area enclosed by a circle of the type shown by the dotted

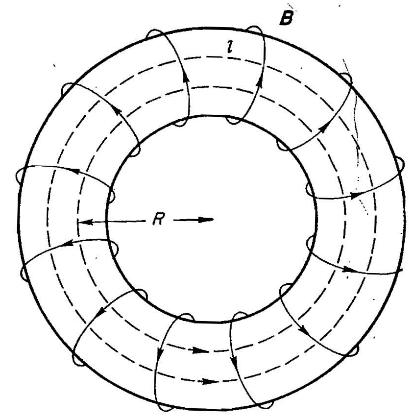

FIG. 4.4. Inhomogeneous axial magnetic field in a torus.

(field) lines in the Fig. 4.4, not necessarily in the median plane of the torus, it follows that, if dA is an element of area,

$$\int (\nabla \times \mathbf{B}) \cdot dA = \frac{4\pi}{c} \int \mathbf{j} \cdot dA = \text{constant},$$

since the integral over $\mathbf{j} \cdot dA$ is constant. The first integral may be written in an alternative form as the integral around the torus of $\mathbf{B} \cdot dl$, where dl is an element of length of the circular path; thus,

$$\int (\nabla \times \mathbf{B}) \cdot dA = \oint \mathbf{B} \cdot dl = \text{constant}.$$

If R is the radius of the path, so that $l = 2\pi R$,

$$\oint \mathbf{B} \cdot dl = 2\pi RB = \text{constant},$$

and hence

$$B = \frac{\text{constant}}{R}. \qquad (4.21)$$

The magnetic field strength is thus inhomogeneous, since its value at any point is inversely proportional to the distance R of that point from the major axis of the torus.

4.35. The total drift velocity of a charged particle, perpendicular to the field lines, is the sum of that due to inhomogeneity of the magnetic field and that due to the curvature of the lines of force. The former contribution is given by equation (4.16), with $\nabla_{\perp}B/B$ equal to $1/R$, since BR is constant, and the latter by equation (4.19), so that

$$|v_d| = \frac{W_{\perp}c}{eBR} + \frac{2W_{\parallel}c}{eBR} = \frac{c(W_{\perp} + 2W_{\parallel})}{eBR}.$$

For a Maxwellian distribution of velocities, $W_{\perp}$ is equal to kT and $W_{\parallel}$ to $\frac{1}{2}kT$ at the kinetic temperature T; hence, the drift velocity may be represented by

$$|v_d| = \frac{2ckT}{eBR}. \tag{4.22}$$

4.36. It should be noted that this expression gives only the absolute magnitude of the total drift velocity. Since the drift directions due to both inhomogeneity and curvature have the same dependence on the sign of the charged particle, it can be readily seen that ions will tend to drift in one direction, perpendicular to the plane of the torus, and electrons will tend to drift in the opposite direction. The resulting space charge will produce an electric field which, acting in conjunction with the magnetic field, will cause the plasma as a whole to drift in a direction perpendicular to both fields, as in §4.19. There is no simple relationship between the drift velocity of the plasma as a whole and the individual particle drift velocities, given by equation (4.22). However, since the magnitude of the space-charge electric field must be related to v_d, it would appear that the plasma drift in a toroidal magnetic field can be reduced by increasing the magnetic field strength and the major radius of the torus.

COLLISION PHENOMENA IN A PLASMA

INTRODUCTION

4.37. In the preceding discussion it has been supposed that collisions among the charged particles are rare; that is to say, the collision mean free path is long compared with the dimensions of the confining field, so that the single-particle picture of a plasma is valid. In the present section various aspects of the collision behavior of charged particles will be examined. Estimates will be made of the mean free path for collisions and the exchange of energy accompanying collisions between charged particles. Scattering collisions and energy changes may be regarded as having a perturbing effect on the single-particle behavior in electromagnetic fields.

4.38. The collision between charged particles differs in a highly important respect from that between neutral particles or between a neutral and a charged particle. In the latter cases, there is a fairly definite collision diameter; whenever two particles are within this distance from each other a collision will have occurred. Any approach of the two particles at distances greater than the collision diameter will not result in any interaction. With two charged particles, on the other hand, the situation is very different because the effective range of the Coulomb force, upon which scattering collisions depend, is infinite. In considering charged-particle collisions, it is necessary first to define exactly what type of encounter is to be regarded as a collision. This is generally taken as the interaction which will lead to a deflection (or scattering) through a large angle, namely, 90° or more.

4.39. For simplicity of treatment, the collisions are divided into two categories, although no such distinction actually exists in a plasma. In the first category are the *short-range encounters* (or close collisions) which lead to a scattering angle of 90° or more in a single interaction between a pair of charged particles. The second type is that of *long-range encounters* (or distant collisions); these represent the multiple interactions of a single particle with many other particles such that the net effect is to give a large-angle scattering, i.e., about 90°. In principle, these long-range encounters can extend over the whole distance over which the Coulomb forces are effective, i.e., the whole of the plasma. However, in order to make possible the calculation of the cross section (or equivalent mean free path) for distant collisions of the type just defined, it is necessary to choose a characteristic distance, called the Debye shielding length, within which interaction of a given charged particle with other charged particles may be supposed to occur. Beyond this distance, the plasma may be regarded as being electrically neutral, both macroscopically and microscopically, so that the particle under consideration is not affected by Coulomb forces [2, 7, 8, 12].

ELECTRICAL NEUTRALITY OF A PLASMA

4.40. A basic property of a plasma, which is a consequence of long-range collective interactions among the charged particles, is the tendency toward electrical neutrality. If, over a relatively large volume of the plasma, the density of electrons should differ appreciably from the positive ion density, large electrostatic forces will come into play. As a result, the charged particles will move rapidly in such a manner as to approach a condition of charge equality.

4.41. Some indication of the order of magnitude of the electrostatic fields that would result from a departure from electrical neutrality over an appreciable volume may be obtained by considering a hydrogen isotope plasma in